Estimating Stock Market Volatility Using Exponential Garch Model with Skewed Student- T Distribution

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Abstract

The aim of the study is to empirically investigate the performance of the EGARCH (1, 1) volatility model with the normal, skew-normal, and student t and skewed student t distributions on the NSE Nifty Fifty Index. Ten years of daily closing rates over the period of January 2010 to December 2020, for a total of 2730 observations, have been analyzed. According to the information criterion, this study has found that the EGARCH (1, 1) model under skewed student t distribution is a better fit than other distribution models.

Keywords: GARCH, EGARCH, Student t distribution, skewed student distribution.

Introduction

Traders use various metrics to determine the relative risk of a potential trade. A highly volatile stock is inherently riskier, but that risk cuts both ways. When investing in a volatile security, the chance for success is increased as much as the risk of failure(Treleaven, 2013).

When asset prices fluctuate sharply over time, differentials are important. The higher the volatility for a given capital structure, the higher the probability of default. The increased risk associated with a given economic activity should, therefore, see a reduced level of participation in that activity, which will have adverse consequences for investment(Daly, Financial volatility: Issues and measuring techniques, 2008).

The primary source of changes in market prices is the arrival of news about an asset’s fundamental value. Some volatility clusters are short-lived, lasting only a few hours, while others last a decade. At lower frequencies, macroeconomic and institutional changes are the most likely influences(Duffie, 2010),(Daly, 2011).

Trading and non-trading days contribute to market volatility. During recessions and financial crises, stock market volatility tends to be high. Leverage effects provide a partial explanation for market volatility changes (Bhowmik, 2013). The measured effects of stock price changes on volatility are too large to be explained solely by leverage changes (Figlewski, 2000).

According to (Engle, 1987), financial market volatility is predictable. The implication for risk-adverse investors is that they can adjust their commitments to assets whose volatilities are predicted to increase, thereby reducing their exposure to risk. Volatility forecasting is an imprecise activity, just like predicting rain.

The most important development in

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modelling volatility changes was the autoregressive conditional Heteroskedasticity, or ARCH, model, introduced by (Officer, 1973). The growth rate of the ARCH literature has been spectacular over the last decade. Numerous applications of ARCH models defy observed trends in scientific advancement.

ARCH techniques have been used to model the relationship between time-varying conditional variance and the risk premium in the term structure of interest rates. In modelling exchange rate dynamics, international portfolio management depends on expected exchange rate movement through time. The linear GARCH (p, q) model has been widely used for modelling exchange rate dynamics. This study focuses on the EGARCH model that allows good and bad news to have a different impact on volatility, while the standard GARCH model does not, and it allows big news to be more impactful in determining how much volatility is affected by economic or political events.

\[ r_t = \ln \frac{l_t}{l_{t-1}} \times 100 \]

The conditional variance equation in the EGARCH model is set as follows:

\[ \ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \alpha |\epsilon_{t-1}/\sigma_{t-1}| - \sqrt{2/\pi} + \gamma(\epsilon_{t-1}/\sigma_{t-1}). \]

The left is the logarithm of conditional variance which means that the lever effect is exponential rather than secondary; so, the predictive value of conditional variance certain is nonnegative. The existence of the lever effect is tested through the hypothesis \( \gamma \neq 0 \). As long as \( \gamma \neq 0 \), the effect of shocks exist in non-symmetries (Hao Liu, 2009).

Method and Materials

Data collection and diagnostic tests

This study uses the daily return of NSE Nifty fifty close price from NSE website (www.nseindia.com) over the period from January 2010 to December 2020. These close prices are converted to daily return. R Studio is used to test the daily return series for various statistical properties such as normality test, QQ plot and GARCH estimation. The relative return on the market index is calculated as:

Econometric models selection and their estimation

This study applied the EGARCH model of order (1, 1) for the volatility estimation and tested with four different distributions such as normal, skew-normal, student t and skewed student t distribution (A, 2022).

(Nelson, 1991) Proposed the E-GARCH model to address a few issues with the GARCH model. This paper explores how asset returns can be affected by valuation asymmetries.

Data Analysis

First, the data’s stationarity is checked using the Augmented Dickey-Fuller Test, and then the ARCH test is used to examine the time dynamics of conditional variance. By analyzing the QQ plot of various distributions for best fit distributions and estimating the EGARCH model under various distributions. Finally, select the best model based on the information criterion and, on the basis of empirical results, the conclusions are drawn.

Analysis and discussions

Table 1 discusses the descriptive statistics of the log return series and clearly shows the presence of larger kurtosis and higher deviation among the various statistical properties. This has significant consequences for analyzing the risk-return relationship and the time-varying nature of correlations.
A unit root test detects if a non-stationary time series variable has a unit root. The null hypothesis is the presence of a unit root, while the alternative hypothesis is stationarity, trend stationarity, or explosive root. Table 2 indicates that the unit root test shows that the first difference between the Nifty Fifty is all stationary.

**Table 2. Augmented Dickey-Fuller Test**

<table>
<thead>
<tr>
<th>Dickey-Fuller</th>
<th>Lag order</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-13.558</td>
<td>13</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The Lagrange Multiplier test introduced by Engle (1982) assesses the significance of a fitted linear regression model for the squared residuals. The null hypothesis, therefore, states that the squared residuals are a sequence of white noise, i.e., that the residuals are homoscedastic. The table 3 LM test shows a p-value less than 0.05, which indicates that the null hypothesis (no arch effect) can be rejected. Therefore, the log of stock returns has an ARCH effect.

**Table 3. ARCH LM-test**

<table>
<thead>
<tr>
<th>Chi-squared</th>
<th>df</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>758.64</td>
<td>12</td>
<td>&lt;2.2e-16</td>
</tr>
</tbody>
</table>

Source: Author own Calculation and Compliance
The daily return plot appears to show a stationary process with a zero mean, but the volatility shows periods of relative calm followed by swings (volatility clustering). The white-noise test reveals that there is no substantial serial correlation in the time series, but the ARCH effect is significant and indicates time-varying volatility (Harvey, 1992).
Figure 3. Quintile plots for log return series of the NSE Nifty fifty, Normal distribution (first row-left), Skew-Normal (first row-right), Student t distribution (Second row-left), and Skewed student t distribution (Second row-right).

The Q-Q plot depicts an asymmetrical view of the distribution tails; the left tail (i.e., extreme negative returns) deviates significantly from the Gaussian distribution. In the financial time series, this is a well-documented phenomenon. The theoretical quantile of a skewed student t distribution shows more fit to the data compared with other distributions. The literature has proposed numerous generalizations of the student’s t distribution in recent years. Applications-wise, the student’s t distribution and its generalizations have become widespread. The most prevalent economic and financial data models (Li, 2020).

Table 4 shows the estimated parameters of EGARCH with different distributions. The results indicate that EGARCH with student t distribution and skewed student t distribution, all parameters are significant.

Table 4. Parameter Estimation for EGARCH (1, 1)
Model under Four distributions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Normal</th>
<th>Skew-Normal</th>
<th>Student t</th>
<th>Skewed Student t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mu</td>
<td>0.000314</td>
<td>0.000256</td>
<td>0.000501</td>
<td>0.000345</td>
</tr>
<tr>
<td></td>
<td>(0.64623)</td>
<td>(0.081161)</td>
<td>(0.005361)</td>
<td>(0.034794)</td>
</tr>
<tr>
<td>Omega</td>
<td>-0.229405</td>
<td>-0.233557</td>
<td>-0.219241</td>
<td>-0.221763</td>
</tr>
<tr>
<td></td>
<td>(0.51850)</td>
<td>(0.000000)</td>
<td>(0.000000)</td>
<td>(0.000000)</td>
</tr>
<tr>
<td>Alpha 1</td>
<td>-0.101163</td>
<td>-0.101067</td>
<td>-0.104408</td>
<td>-0.104399</td>
</tr>
<tr>
<td></td>
<td>(0.68921)</td>
<td>(0.000070)</td>
<td>(0.000000)</td>
<td>(0.000000)</td>
</tr>
<tr>
<td>Beta 1</td>
<td>0.974986</td>
<td>0.974482</td>
<td>0.976700</td>
<td>0.976223</td>
</tr>
<tr>
<td></td>
<td>(0.00000)</td>
<td>(0.000000)</td>
<td>(0.000000)</td>
<td>(0.000000)</td>
</tr>
<tr>
<td>Gamma 1</td>
<td>0.119668</td>
<td>0.118766</td>
<td>0.111104</td>
<td>0.112046</td>
</tr>
<tr>
<td></td>
<td>(0.15133)</td>
<td>(0.000000)</td>
<td>(0.000000)</td>
<td>(0.000000)</td>
</tr>
<tr>
<td>Skew</td>
<td>0.898085</td>
<td>0.906399</td>
<td>0.906399</td>
<td>0.906399</td>
</tr>
<tr>
<td></td>
<td>(0.00000)</td>
<td>(0.000000)</td>
<td>(0.000000)</td>
<td>(0.000000)</td>
</tr>
<tr>
<td>Shape</td>
<td></td>
<td></td>
<td>8.222721</td>
<td>8.426183</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.000000)</td>
<td>(0.000000)</td>
</tr>
</tbody>
</table>

Source: Author own Calculation and Compliance
Note. The first row indicates estimate values and the second row shows p-values

Table 6 explains that the task of selecting one model from a group of candidates is known as “model selection.” The Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) were used to choose appropriate models (BIC). Trial and error tests were carried out using the R statistical software to find the best-fitting model. The goal was to create a compact model that captured as much variation in the data as possible. The AIC and BIC values of the best model should be lower. Models with higher AIC and BIC values are more effective (West, 2012).

It is thought to be inappropriate. Of the two approaches used in the selection, the EGARCH model had the most negative values. The fitted models with the minimum AIC and BIC values are the EGARCH model with a skew student t distribution.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Normal</th>
<th>Skew-Normal</th>
<th>Student t</th>
<th>Skewed Student t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akaike</td>
<td>-6.4896</td>
<td>-6.4963</td>
<td>-6.5180</td>
<td>-6.5222</td>
</tr>
<tr>
<td>Bayes</td>
<td>-6.4788</td>
<td>-6.4833</td>
<td>-6.5050</td>
<td>-6.5070</td>
</tr>
</tbody>
</table>

**Conclusion**

This study compared the performance of the EGARCH model using normal, skew-normal, student t, and skewed student t distributions on the daily returns of the NSE Nifty Fifty over a 10-year period. The left tail of the log of stock returns deviates significantly from the Gaussian distribution, exhibiting an ARCH impact. Compared to other distributions, the theoretical quantile of a skewed student t distribution fits the data better. The objective was to develop a compact model that caught as much data variance as feasible.

Empirical results demonstrate that incorporation of EGARCH with a skewed student t distribution gives proper volatility estimation under the circumstances of skewness and heavy tail in the data (Alberg, 2008).

Estimating and forecasting high-frequency financial time series using asymmetric GARCH models can improve the outcomes. As long as non-normal densities are utilized with obviously non-normal series, adding non-normal densities is a potential field. These are the non-central Student-t and skewed Student-t distributions, as suggested in the context of strong stochastic volatility (Xie, 2021).

**References**


