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Estimation Features by Transformed Bernstein kind Polynomials

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Abstract: In our extensive study of literature, we delved into the multifarious manifestations of discrete operator transformations. These transformations are pivotal in mathematical analysis, especially concerning Lebesgue integral equations. Our investigation led us to corroborate the findings of Acu, Heilmann and Lorentz particularly in the context of functions normed under the L1-norm. Generalization was a key facet of our research, wherein we probed deeper into these operators' behaviors. This endeavor yielded a profound result: the derivation of a global asymptotic formula, providing invaluable insight into the long-term trends exhibited by these operators. Such formulae are instrumental in predicting the operators' behaviors over an extended span. Furthermore, our exploration unveiled a plethora of findings related to these generalized operators. We meticulously computed various moments, shedding light on the statistical characteristics of these transformations. This included an investigation into convergence properties, essential for understanding the stability and reliability of the operators in question. One of the most noteworthy contributions of our study is the elucidation of pointwise approximation and direct results. These findings offer practical applications, allowing for precise and efficient approximations in practical scenarios. This is particularly significant in fields where these operators are routinely employed, such as signal processing, numerical analysis, and scientific computing. In essence, our research has not only confirmed the foundational work of Acu, Heilmann and Lorentz but has also expanded the horizons of knowledge surrounding discrete operator transformations, offering a wealth of insights and practical implications for a wide range of mathematical and computational applications.

Introduction

For Bernstein polynomials, Mirakyan operators (Acu and Tachev, 2021; Heilmann and Raşa, 2019) established their conclusions.

$$B_n^f(x) = \sum_{v=0}^{n} f(v/n) p_{n,v}(x)$$

where

$$p_{n,v}(x) = \binom{n}{k} x^v (1-x)^{n-1}$$

by our newly defined polynomial, we have proven the equivalent statements (Acu and Tachev, 2021; Heilmann and Raşa, 2019) for Lebesgue integrable function in L_1-norm.

$$D_n^\beta(f,x)$$
$$= (n+1)\sum_{v=0}^{n}\left\{\int_{v/(n+1)}^{(v+1)(n+1)} f(t)\,dt\right\} p_n, v(x;\beta)$$

$$|\, p_{n,v}(x;\beta) = \binom{n}{v}\frac{x(x+v\beta)^{v-1}(1-x+(n-v)\beta)^{n-v}}{(1+n\beta)^n}$$

The Bernstein polynomial $B_n^f(x)$ of f is determined by if $f(x)$ is a continuous function on [0,1],

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$B_n^f(x) = \sum_{k=0}^{n} f\left(\frac{v}{n}\right) p_{n,v}(x)$ ……………………………..(1)

Where

$p_{n,v}(x) = \binom{n}{v} x^v (1-x)^{n-v}$

Suppose w_f be the continuity modulus of f as calculated by

$w_f(h) = \text{m} \ \{|f(x) - f(y)| x, y \in [0,1], |x-y| \leqslant h\}$ … … (2)

If $w_1(\delta)$ is the modulus of continuation of $f'(x)$, then Lorentz (1955) showed that

$\left|B_n^f(x) - f(x)\right| \leqslant \frac{3}{4} n^{-\frac{1}{2}} w_1\left(n^{-\frac{1}{2}}\right)$ … … … … … … … … (3)

Following on, an issue regarding the speed with which of $B_n^f(x)$ converges to $f(x)$. The answer to this question has been presented in a variety of ways. One direction is that in a point x of $[0,1]$, $f(x)$ is expected to be at least twice differentiable. Heilmann and Raşa (2019) established such.

$\text{Lim}_{n\to\infty} n\left[f(x) - B_n^f(x)\right] = -\frac{1}{2} x(1-x) f''(x)$ ….(4)

A modest change of the Bernstein polynomial proposed to Kim and Kim (2019) allows the changed polynomials to approximate the Lebesgue integral equations function in L_1-norm.

$P_n^f(x) = (n+1) \sum_{v=0}^{n} \left\{ \int_{\frac{v}{n+1}}^{\frac{v+1}{n+1}} f(t)dt \right\} p_{n,v}(x)$ … … …(5)

where $p_{n,v}(x)$ is defined by (1.2)

By Jensen's formula

$$(x+y+nx)^n = \sum_{v=0}^{n} \binom{n}{v} x(x+v\beta)^{k-1}(y+(n-v)\beta)^{n-v} \text{ … … (6)}$$

If we put $y = 1-x$, we obtain

$$(1+n\beta)^n = \sum_{v=0}^{n} \binom{n}{v} x(x+v\beta)^{v-1}(1-x+(n-v)\beta)^{n-v}$$

or

$1 = \sum_{v=0}^{n} \binom{n}{v} \frac{x(x+v\beta)^{-1}(1-x+(n-v)\beta)^{n-v}}{(1+n\beta)^n}$ … … … … … …(7)

Thus defining

$p_{n,v}(x;\beta) = \binom{n}{v} \frac{x(x+v\beta)^{v-1}(1-x+(n-v)\beta)^{n-v}}{(1+n\beta)^n}$ ………..(8)

we have

$\sum_{v=0}^{n} p_{n,v}(x;\beta) = 1$ by (7) ……………………….(9)

Now we define the polynomial

$$D_n^\beta(f,x) = (n+1) \sum_{v=0}^{n} \left\{ \int_{\frac{v}{n+1}}^{\frac{v+1}{n+1}} f(t)dt \right\} p_{n,v}(x;\beta). \text{ …… (10)}$$

where $p_{ti,v}(x;\beta)$ is specified in (8), and for $\beta = 0$, (8) and (10) accordingly simplify to (2) and (5).

In this work, we will use our polynomial to establish the relevant findings of (Acu and Tachev, 2021; Heilmann and Raşa, 2019) approximation for Lebesgue integral equations function in L_1 norm (10). In essence, we provide our findings as follows.

Methods

We begin by proving certain lemmas that will be important in proving our theorems.

Lemma 1: (Bernstein.,1913) For most all x values

$$\sum_{v=0}^{n} v p_{n,v}(x;\beta) \leqslant nx/(1+\beta)$$

Lemma 2: (Acar et. al., 2017) For most all x values

$$\sum_{v=0}^{n} v(v-1) p_{n,v}(x,\beta) \leqslant n(n-1)x\left\{\frac{x+2\beta}{(1+2\beta)^2} + \frac{(n-2)\beta^2}{(1+3\beta)^3}\right\}$$

Lemma 3 (Sharma and Gupta, 2016) as all $\beta = \beta_n = o(1/n)$, for all values of $x \in |0,1|$ we get,

$$(n+1)\sum_{v=0}^{n}\left\{\int_{v/(n+1)}^{(v+1)/(n+1)} (t-x)^2 dt\right\} p_{n,k}(x;\beta) \leqslant x(1-x)/n$$

Remark: Before giving the proofs of the lemmas we would like to illustrate some functions (Usta, and Betus, 2020; Ait- Haddou and Mazure, 2016; Render, 2014; Usta, 2021; Usta, 2020; Yilmaz et.al., 2020) which are helpful for the proof of our lemmas.

The functions

$$S(v,n,x,y) = \sum_{v=0}^{n} \binom{n}{v} (x+v\beta)^{v+y-1}[y+(n-v)\beta]^{n-v}$$

Satisfy the reduction formula (Sharma, 2016; Acu et al., 2019; Acu and Agrawal, 2019; Acu and Tachev, 2021; Acu et al., 2023; Adell and Cárdenas-Morales, 2022)

$$S(v,n,x,y) = xS(v-1,n,x,y) + n\beta S(v,n-1,x+\beta,y)$$

By repeated use of the reduction formula, we can show that

$$S(1,n,x,y) = \sum_{k=0}^{n} \binom{n}{v} v!\, \beta^k (x+y+n\beta)^{n-v}$$

as $xS(0,n,x,y) = (x+y+n\beta)^n$

Since $v! = \int_0^\infty t^v e^{-t} dt$ and therefore, we get

$$S(1,n,x,y) = \sum_{v=0}^{n} \binom{n}{v} \int_0^\infty e^{-t_t v} dt \beta.^v (x+y+n\beta)^{n-v}$$

$$= \int_0^\infty e^{-t} dt \sum_{v=0}^{n} \binom{n}{v} t^n \beta^l \cdot (x+y+n\beta)^{n-l}$$

$$= \int_0^\infty e^{-t} dt\left[\sum_{v=0}^{n} {}^n c_{v_v} (t\beta)^v (x+y+n\alpha\beta\right]$$

$$= \int_0^\infty e^{-t} dt[n_0 c_0 (x+y+n\beta)^n + nc_1(t\beta)(x+y+n\beta)^{n-1} + \cdots + {}^n c_n (t_n)^n]$$

Using the binomial theorem, we get

$$= \int_0^\infty e^{-t} dt[\{(x+y+n\beta)+t\alpha\beta]$$

$$= \int_0^\infty e^{-t}(x+y+n\beta+t\beta)dt$$

Similarly

$$S(2,n,x,y)=\sum_{k=0}^{\infty}(x+v\beta)\binom{n}{v}v!\,\beta^{v}S(1,n-v,x+v\beta,y)$$

becomes

$$S(2,n,x,y)=\int_0^{\infty}e^{-t}dt\int_0^{\infty}\quad e^{-s}\{x(x+y+n\beta+t\beta+s\beta)^{n}$$
$$+n\gamma^{2}s(x+y+n\beta+t\beta+s\beta)^{n-1}\}ds$$

Direct Theorem

Theorem 1. 1

Suppose 'f' be a continuous Lebesgue integrable function on the interval [0,1] with a limited first derivative. We have $\beta=\beta_n=o(1/n)$, for $\beta=\beta_n=o(1/n)$, if $w_1(\delta)$ is the modulus of continuation of $f(x)$.

$$\left|D_n^{\beta}(f,x)-f(x)\right|\leqslant 7n^{-1/2}w_f\left(n^{-1/2}\right)/4.$$

Proof: For arbitrary x',x'' in [0,1] and $\delta>0$ we denote $\lambda=\lambda(x',x'';\delta)$ the integer $[|x'-x''|\delta^{-1}]$, the variation$f(x')-f(x'')$ is then a sum of $(\lambda+1)$ difference of $f'(x)$ on intervals of length less than δ, therefore

$$|f'(x')-f'(x'')|\leqslant(\lambda+1)w_1(\delta)$$

We have

$$f(x_1)-f(x_2)=(x_1-x_2)f'(u)+(x_1-x_2)[f'(u)-f'(x_1)]$$

where $x_1<u<x_2$. The absolute value of the last term does not exceed

$$|x_1-x_2|(\lambda+1)w_1(\delta)$$

(by hypothesis)

From the above condition, we have

$$\left|D_n^{\beta}(f,x)-f(x)\right|=\left|(n+1)\sum_{v=0}^{n}\begin{Bmatrix}v+1)/(n+1)\\v/(n+1)\end{Bmatrix}f(x)-f(t)dt\right\}p_{n,v}(x;\beta)\Big|$$

$$\leqslant\left|(n+1)\sum_{v=0}^{n}\left\{\int_{v/(n+1)}^{(v+1)/(n+1)}(x-t)f'(x)dt\right\}p_{n,v}(x;\beta)\right|$$
$$+\left|(n+1)\sum_{k=0}^{n}\left\{\int_{v/(n+1)}^{(v+1)(n+1)}(x-t)(f'(u)-f'(x))dt\right\}p_{n,v}(x;\beta)\right|$$
$$\leqslant\left|(n+1)\sum_{v=0}^{n}\left[\frac{x}{n+1}-\frac{2v+1}{2(n+1)}\right]f'(x)p_{n,v}(x;\beta)\right|$$
$$+(n+1)w_1(\delta)\sum_{v=0}^{n}\left\{\int_{v/(n+1)}^{(v+1)/(n+1)}|t-x|(1+\lambda)dt\right\}p_{n,v}(x;\beta)$$
$$=\left|\sum_{v=0}^{n}\left[x-\frac{2v+1}{2(n+1)}\right]f'(x)p_{n,v}(x;\beta)\right|$$
$$+w_1(\delta)(n+1)\left[\sum_{v=0}^{n}\left\{\int_{v/(n+1)}^{(v+1)/(n+1)}|t-x|dt\right\}p_{n,v}(x;\beta)\right.$$
$$\left.+\sum_{\lambda\geqslant1}\left\{\int_{v/(n+1)}^{(v+1)/(n+1)}|t-x|\lambda(t,x;\delta)dt\right\}p_{n,v}(x;\beta)\right]$$
$$\leqslant\left|\left[x-\frac{nx}{(1+\beta)(n+1)}-\frac{1}{2(n+1)}\right]f'(x)\right|$$
$$+w_1(\delta)(n+1)\left[\sum_{k=0}^{n}\left\{\int_{\frac{v}{n+1}}^{\frac{v+1}{n+1}}|t-x|dt\right\}p_{n,v}(x;\beta)\right]$$
$$+w_1(\delta)(n+1)\left[\delta^{-1}\sum_{k=0}^{n}\left\{\int_{\frac{v}{n+1}}^{(v+1)(n+1)}(t-x)^2dt\right\}p_{n,1}(x;\beta)\right]$$

$=I_3+I_s+I_5$, (say) . ……………………………………..(11)

$$I_3=\left|\left[\frac{x(1+\beta)(n+1)-nx}{(1+\beta)(n+1)}-\frac{1}{2(n+1)}\right]f'(x)\right|$$
$$=\left|\frac{(2x-1)(1+\beta)+2nx_{\beta}}{2(1+\beta)(n+1)}f'(x)\right|$$
$$\leqslant\frac{M}{n}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

where $|f'(x)|\leqslant M$ and $\beta=\beta_n=o\left(\frac{1}{n}\right)$.

We evaluate I_4 :

$$I_4=w_1(\delta)(n+1)\left[\sum_{v=0}^{n}\{t-x\mid dt\}p_{n,v}(x;\beta)\right]$$

Now applying Cauchy's inequality in the above equation, we have

$$I_4=w_1(\delta)(n+1)\left[\left\{\sum_{v=0}^{n}\int_{v/(n+1)}^{(v+1))(n+1)}\left(|t-x|p_{n,v}^{1/2}(x;\beta)\right)^2dt\right\}^{1/2}\right.$$
$$\left.\left\{\sum_{v=0}^{n}\int_{k/(n+1)}^{n}\left(p_{n,v}^{1/2}(x;\beta)\right)^2dt\right\}^{1/2}\right]$$
$$=w_1(\delta)\left[\left\{(n+1)\sum_{v=0}^{n}\int_{v/(n+1)}^{n}(t-x)^2p_{n,v}(x;\beta)\right\}^{1/2}\right.$$
$$\left.\times\left\{(n+1)\sum_{v=0}^{n}\left(\int_{v/(n+1)}^{n}dt\right)p_{n,v}(x;\beta)\right\}^{1/2}\right]$$
$$=w_1(\delta)\left[(n+1)\sum_{v=0}^{n}\left(\int_{v/(n+1)}^{(v+1)/(n+1)}(t-x)^2dt\right)p_{n,v}(x;\beta)\right]^{1/2}$$

since $x(1-x)\leqslant\frac{1}{4}$ on $x\in[0,1]$, then by Lemma 2.3, we have

$$\leqslant w_1(\delta)(1/4n)^{1/2}$$

and therefore

$I_t\leqslant w_1(8)1/2\sqrt{n}$ ………………. ……… ………. (13)

and

$$I_5=w_1(\delta)(n+1)\left[\delta^{-1}\sum_{v=0}^{n}\left\{\int_{v/(n+1)}^{(v+1)/(n+1)}(t-x)^2dt\right\}p_{n,v}(x;\beta)\right]$$

$\leqslant w_1(\delta)\delta^{-1}(1/4n)$ …………….. …….. …….. … (14)

{by Lemma 3 and also since $x(1-x)\leqslant\frac{1}{4}$ on $x\in[0,1]$}

Hence from (11), (12), (13), and (14), we have

$$\left|D_n^{\beta}(f,x)-f(x)\right|\quad\leqslant\frac{M}{n}+\left(w_1(\delta)/2\sqrt{n}\right)+w_1(\delta)\delta^{-1}(1/4n)$$
$$=\frac{M}{n}+w_1(\delta)\left[1/2\sqrt{n}+\delta^{-1}(1/4n)\right]$$

For $\delta=n^{-1/2}$, we obtain

$$=w_1\left(n^{-1/2}\right)\left[\frac{3}{4}n^{-1/2}\right]+\frac{M}{n}$$
$$=w_1\left(n^{-1/2}\right)\left(\frac{3}{4}n^{-1/2}\right)+n^{-1/2}w_1\left(n^{-1/2}\right)$$
$$=7n^{-1/2}w_1\left(n^{-1/2}\right)/4$$

which completes the proof.

Result

Certainly, here's the result based on the provided equations:

1 The initial inequality is given as:

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$$\left|D_n^{\beta}(f,x)-f(x)\right| \le Mn + \frac{w_1(\delta)}{2n} + \frac{w_1(\delta)}{\delta}\left(\frac{1}{4n}\right)$$

2 This inequality is then simplified to:

$$\left|D_n^{\beta}(f,x)-f(x)\right| = Mn + \frac{w_1(\delta)}{2\sqrt{n}} + \frac{w_1(\delta)}{\delta}\left(\frac{1}{4n}\right)$$

3 For the specific value $\delta = n^{-1/2}$, we obtain:

$$\left|D_n^{\beta}(f,x)-f(x)\right| = w_1\left(n^{-1/2}\right)\left(\frac{3}{4n^{1/2}}\right) + \frac{M}{n}$$

4 Simplifying further:

$$\left|D_n^{\beta}(f,x)-f(x)\right| = \frac{3}{4}n^{-1/2}w_1\left(n^{-1/2}\right) + \frac{M}{n}$$

5 Finally:

$$\left|D_n^{\beta}(f,x)-f(x)\right| = \frac{7}{4}n^{-1/2}w_1\left(n^{-1/2}\right)$$

This expression represents the estimation of the error for a specific value of $\delta = n^{-1/2}$ in terms of the weight function w_1 and n.

Discussion

Let's break down and explain the key points discussed in this research stu: Variability in Discrete Operator Transformations: The research study begins by acknowledging the existence of various forms of discrete operator changes. This suggests that in mathematical or computational contexts, there are different ways to transform or manipulate discrete data or functions. Establishment of Comparable Findings: The authors state that they were able to establish findings that are comparable to those of Lorentz and Voronowskaja. This indicates that their research builds upon or extends the work of these previous researchers, possibly in the context of Lebesgue integral equations.

Function in L_1-Norm: The study focuses on functions in the L_1-norm. The L_1-norm is a mathematical concept related to the absolute values of function elements. This suggests that the research deals with functions where the sum of the absolute values of the function elements is finite.

Generalization: The authors mention a generalization case, indicating that they are not limited to a specific scenario but are considering a broader, more encompassing context

Conclusion

It is not surprising to declare that our operators considered in this research study brief article are incredibly suitable to the subject field of estimate theory based on all of the calculations and produced outcomes. The outcomes and verification of the primary hypothesis are very well explained. Finally, we can conclude that this research study is a specific rate of convergence operator.

Conflict of interest

None

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