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Impact of Auto-regressive (AR) Process in Bullwhip Analysis in a Multi-location Supply Chain Network

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Abstract: The present study is an attempt to quantify the Bullwhip Effect (BWE) -the phenomenon in which information on demand is distorted in moving up a supply chain. Assuming that the retailer employs an order-up-to level policy with auto-regressive process (AR), the paper investigates the influence of forecasting methods on bullwhip effect. Determining the order-up-to levels and the orders for the retailers' demands in an isolated manner neglects the correlation of the demands and the relevant risk pooling effects associated with the network structure of the supply chains are disregarded. It is illustrated that the bullwhip effects are significantly reduced with consideration of potential correlation between the retailers' demand.

Keywords: *Bullwhip Effect, Demand forecasting, Auto-regressive process*

I. Introduction

The phenomenon of increase in demand variability of the product as one moves from end customers to upstream players such as distributors, manufacturers, and suppliers in a supply chain network is widely recognized and referred to as the *Bullwhip or Whiplash* effect. It can misguide capacity plans and miss production schedules without being able to see the sales of its products at the distribution channel stage when a supply chain plagued with a bullwhip effect. It leads to much inefficiencies: insufficient or excessive capacities, excessive inventory investment, poor customer service as a result of unavailable products or long backlogs, lost revenues, uncertain production planning and high costs of corrections.

Many researchers contributed to quantify the bullwhip phenomenon in the supply chain area (Caplin, 1985; Lee *et. al*., 1997; Baganha and Cohen, 1998; Dejonckheere *et. al*., 2003). Lee *et al*. (1997) identifies the various causes of demand variance for propagating up a supply chain and demonstrates that the supplier's demand variance depends on the alignment of the retailer's orders. Shortage gaming, where retailers inflate their orders to receive a better allocation leading to bullwhip effect was studied by Cachon and Lariviere (1996). The other causes leading to bullwhip effect like demand updating (the supplier is unaware of true retailer demand and so must rationally assume a higher variance), and price fluctuations (retailers purchase more than their short term needs to take advantage of temporary price discounts) were studied by Drezner *et. al*. (1996) and Chen *et al*. (1997). Shapiro and Byrnes (1992) empirically examined the demand variance in the medical supply industry and observed that orders from hospitals exhibit dramatic variability.

Even though some researchers have been conducted to quantify the bullwhip effect, there is still much work to be accomplished in this line of research. In the cases where demand can be forecasted by moving averages models, previous researchers focused mainly on the MA(3) processes (Sucky, 2009), Although Hong and Ping (2007) took into consideration the use of general moving averages forecasting technology, when they investigated the influence to the forecasting methods on bullwhip and carried out rules decreasing the bullwhip effect in a simple two stage supply chain consisting of a single retailer and a single manufacturer, no explicit expression for measuring the bullwhip in supply network with multiple retailers has been derived in their research. The present study focuses on analyzing bullwhip effect for supply chains that possess a network structure. In practice, supply chains can be considered as the networks of geographically dispersed facilities.

2. Problem Definition

Consider a three stage supply chain network consisting of a single distribution centre (DC) supplied from vendors and ‘n’ multiple retailers that face stochastic demands from the customers (Figure 1). Retailers seeks to satisfy customer orders from their stock-on-hand (SOH) follows an (R, *S*) inventory policy, in which the Inventory Position (*IP*) at each retailer is reviewed each day (i.e. R = 1) and an order of a fixed batch size Q_r is placed on the DC to raise the IP to the desired stock level, *S*. It is assumed that the customer demands at retailers are independent over time and identically distributed random variables. Assuming that a review is made at the start of each day, under an order-up-to level policy the *j*th retailer with desired inventory level $S_{j,t}$ places an order $Q_{j,t}$ to the distribution centre in time *t*. The distribution centre in return places an order of quantity $Q_t = \sum_{j=1}^{n} Q_{j,t}$ in time *t* to the vendor.

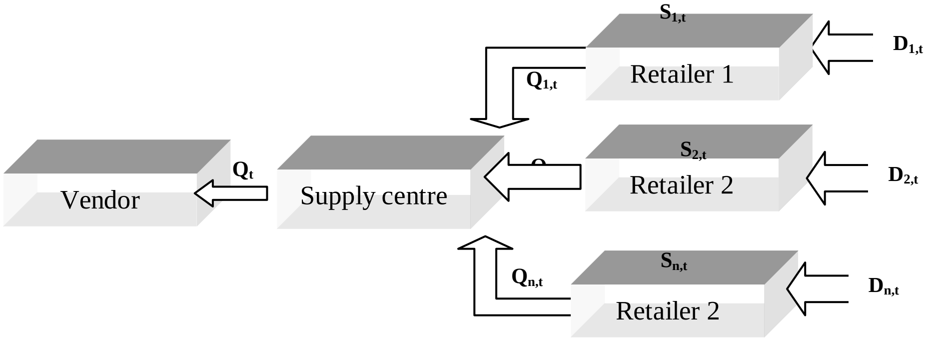


Figure 1**: Three Stage Supply Chain network**

In order to determine the orders Q_t, similar to Chen et al. (2000), we assume that the retailer follows an order-up-to inventory policy. The goal of this ordering policy is to bring the actual inventory towards the desired inventory $S_{1,t}$, $S_{2,t}$,…, $S_{n,t}$ at retailer 1,2,.., n respectively. The order quantity $Q_{1,t}$, $Q_{2,t}$,…, $Q_{n,t}$ the retailer places to the supply centre at the start of period t, is given by

$$Q_{j,t} = S_{j,t} + S_{j,t-1} + D_{j,t-1} \text{ where j = 1,2,…n……………………………….(1)}$$

And the desired target stock levels $S_{j,t}$ with j = 1,2,…,n in period t is estimated from the observed demand as

$$S_{j,t} = E(D_{j,t}) + Z\sqrt{Var(D_{j,t})} \text{ ……………………………………………(2)}$$

Where $E(D_{j,t})$ are estimate of the demand, $\sqrt{Var(D_{j,t})}$ an estimate of the standard deviation of the *j*th retailer's demand, and z ≥ 0 is a managerial determined safety factor chosen to meet a desired level of service (**Zinn *et. al.*, 1989**). For Z = 0 the decision maker is neutral, and z > 0 the decision maker is risk averse.

3. **Bullwhip effect under AR**

Auto-regressive (AR (*p*)) forecasting model is unique, because it does not construct either a single-equation or a simultaneous-equation model; but instead analyzes the probabilistic, or stochastic, properties of the demand process. Unlike the regression models, in which $E(D_{j,t})$ at any j^{th} retail location is explained by *k* explanatory variables, the autoregressive demand process of order *p*, the expected demand, $E(D_{j,t})$ is expressed by a weighted average of the past demands going back *p* periods, together with a random disturbance in the current period.

The expected demands $E(D_{j,t})$ following a *p*-order auto-regressive, AR(*p*) stochastic process can be expressed as:

$$\textbf{AR}(\boldsymbol{p})\text{:}\ E(D_{j,t}) = \alpha_0 + \alpha_1 D_{j,t-1} + \alpha_2 D_{j,t-2} + \ldots + \alpha_p D_{j,t-p} + \varepsilon_t \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Where $\alpha_0, \alpha_1, \ldots, \alpha_p$ are the coefficients to be estimated and ε_t is an error term at time *t* that represents the effects of variables not explained by the model; the assumptions about the error term are the same as those for the Classical Linear Regression Model (CLRM).

The model represented in equation (3) has the appearance of a regression model with lagged values of the dependent variable in the independent variable position, hence the name *auto-regressive model.* Auto-regressive models are appropriate for stationary time series, and the coefficient α_0 is related to the constant level of the series. If the data vary about zero or are expressed as deviations from the mean δ i.e. $E(D_{j,t}) - \delta$, the coefficient α_0 is not required.

Similar to the case of MA and EWMA forecasting methods for order-up-to level policy, order quantity $Q_{j,t}$ to the DC from j^{th} retailer in period *t* using estimated demands through AR(*p*) process (replacing equation (15) by equation (1) and (2) is expressed as:

$$\begin{aligned}
Q_{j,t} &= E(D_{j,t}) + z\sqrt{Var(D_{j,t})} - E(D_{j,t-1}) - z\sqrt{Var(D_{j,t-1})} + D_{j,t-1} \\
&= E(D_{j,t}) - E(D_{j,t-1}) + z\sqrt{Var(D_{j,t})} - z\sqrt{Var(D_{j,t-1})} + D_{j,t-1} \qquad \ldots\ldots\ldots\ldots\ (4) \\
&= D_{j,t-1} + \sum_{i=1}^{p} \alpha_i (D_{j,t-i} - D_{j,t-i-1}) + z\left(\sqrt{Var(D_{j,t})} - \sqrt{Var(D_{j,t-1})}\right)
\end{aligned}$$

Similarly, we suppose that the retailer ignores the security stock inventory (z=0), variances of the order quantity at retailer '*j*' can be estimated as:

$$V(Q_{j,t}) = Var(D_{j,t-1}) + \alpha_1^2 (Var(D_{j,t-1}) + Var(D_{j,t-2})) + ... + \alpha_p^2 (Var(D_{j,t-p}) + Var(D_{j,t-p-1})) \ldots\ldots\ldots \quad (5)$$

However, assuming demands follow a stationary process, then

$$V(Q_{j,t}) = Var(D_{j,t})\left(1 + 2\alpha_1^2 + 2\alpha_2^2 + ... + 2\alpha_p^2\right) \ldots\ldots\ldots \quad (6)$$

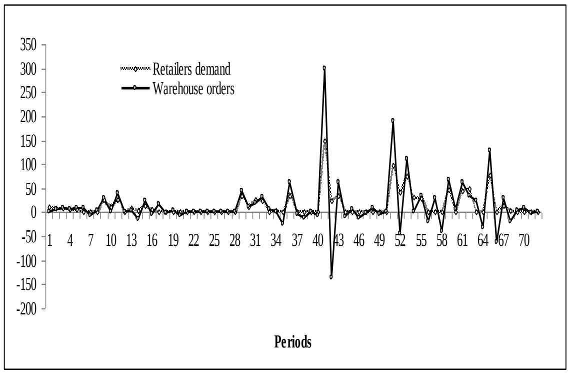
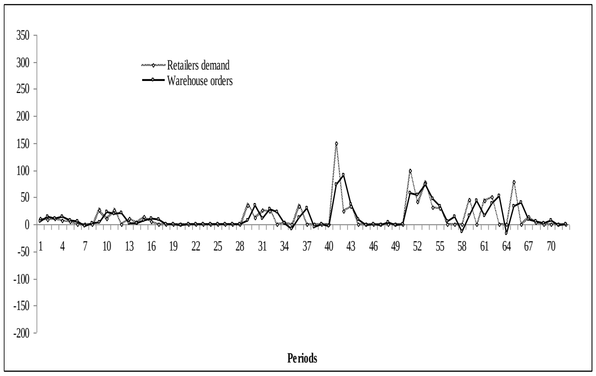
Similar to equation (14), calculating the bullwhip without consider the risk pooling effect

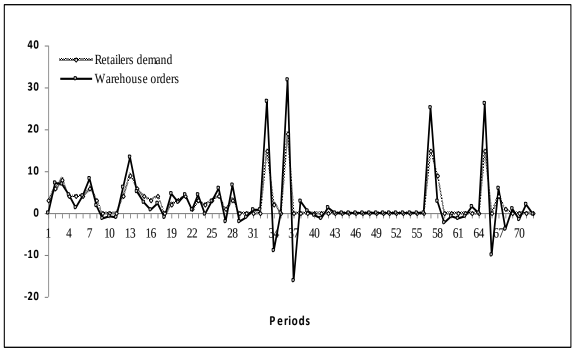
$$Var(Q)/Var(D) = \left(1 + 2\alpha_1^2 + 2\alpha_2^2 + ... + 2\alpha_p^2\right) > 1 \ldots\ldots\ldots \quad (7)$$

4. Data Collection and Analysis

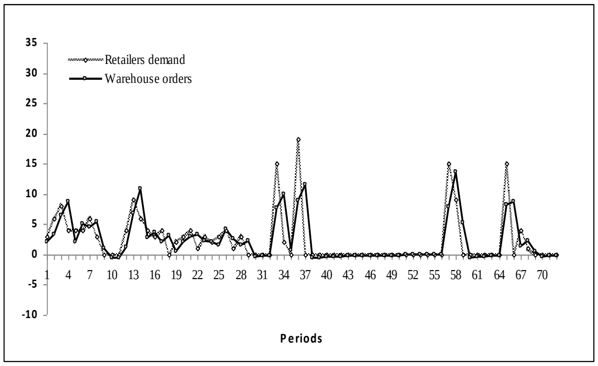
A numerical study based on a two-echelon supply chain network consisting of single distributor (DC) supplied from the vendors and supplying to ten retail stores located geographically at different places is considered. Data about the daily demands of the different packaged (branded) and non-packaged (unbranded) of the end customer at the retail stores are collected. In the initial situation, the retailer determines his orders separately based on the individual demand of each retailer, i.e. without consideration of a potential correlation between the retailers' demands. In the simple situation, the overall order-up-to level under AR(*p*). The calculations are performed on Microsoft Excel spreadsheet. Figure 2 shows the shows that the wholesaler's order quantity varying more than the retailers' periodical demands under different forecasting schemes for the different packaged and non-packaged category of items.

Figure 2: **Order Quantity vs. Retailers' Demand**

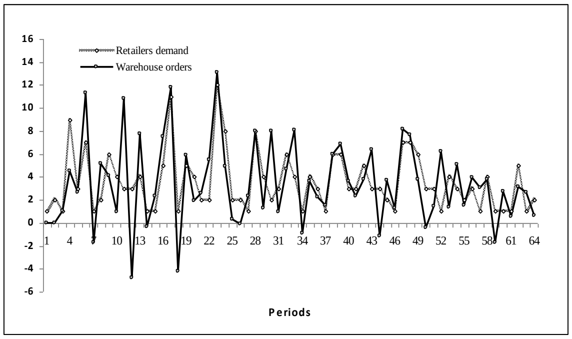
Without Risk Pooling	**With Risk Pooling**
	
PACKAGED ITEM-I	**PACKAGED ITEM-I**



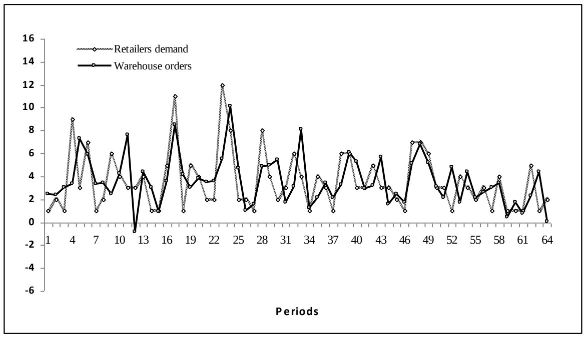
PACKAGED ITEM-II



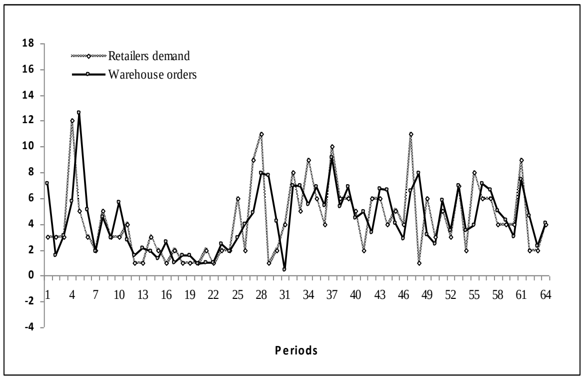
PACKAGED ITEM-II



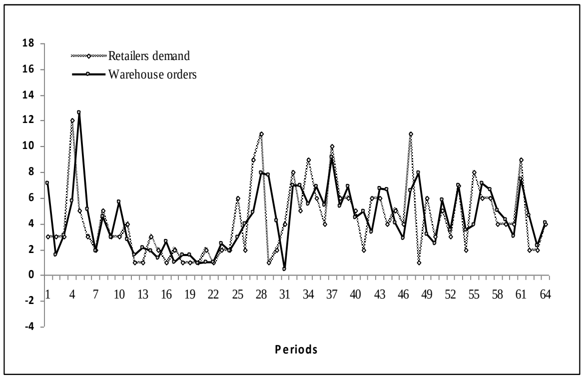
UNPACKAGED ITEM-I



UNPACKAGED ITEM-I



UNPACKAGED ITEM-II



UNPACKAGED ITEM-II

In order to reveal the influence on the bullwhip effects with three-stage supply chain network under different forecasting techniques, the simulated design using different packaged (branded) and non-packaged (unbranded) category of Fast Moving Consumer Goods (FMCG) is employed. Bullwhip measure between the warehouse's order and the retailers' aggregated demands was found to be lower in the non-packaged category of products in comparison to the packaged items (see Table 1).

Determining the order-up-to levels and the orders for the retailers' demands in an isolated manner neglects the correlation of the demands. Therefore, computing the estimated variances with consideration of risk pooling in the retailers' demand could reduce the variability in the warehouse's order and thus reducing the bullwhip. From the results obtained under different categories of products, it is validated that risk pooling has pulled down the bullwhip effect significantly. Interestingly, AR process with consideration of risk pooling has brought down the bullwhip significantly measure to less than one in each category of items considered for the study (see Table 1).

Table 1**: Bullwhip Measures**

Product Category	BWE without risk pooling	BWE with risk Pooling
Soft drinks (Packaged/branded item)	4.098	0.661
Detergent bar (Packaged/branded item)	3.245	0.582
Pulses (Non-packaged/unbranded item)	2.071	0.666
Sugar (Non-packaged/unbranded item)	2.140	0.763

9. Conclusion

The present works shows that bullwhip exists (variance amplification ratio greater than one) for different forecasting technique used for generating desired target levels. It is also concluded that the strength of the bullwhip effect depends on the statistical correlation of the regarded demand. The results based on the established model have also demonstrated the bullwhip effect theory model of the three stage supply chain network on the basis of Auto-Regressive (AR) forecasting methods. Results indicate that the bullwhip due to implementation of AR was better in non-packaged (unbranded) category in comparison to packaged items.

Further, considering the potential correlation in the demands among the retail stores with an idea of risk pooling at the centralized distribution centre, a significant decrease in variability of demand in the upstream was observed under some specific forecasting methods applied in the study. AR processes with risk pooling of retailers' demand at the centralized distribution centre these processes has brought down the bullwhip measure to less than one in all category of items considered in the study.

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