Dynamic Characteristics Analysis of Mold Oscillation Mechanism Generating Non-Sinusoidal Waves

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Abstract

In this study, to understand the vibration characteristics of the mold vibrator, the vertical vibration, including the rotational transmission of a four-bar mechanism generating non-sinusoidal waves, was numerically implemented. Physical characteristics were compared through behavior comparison with a mechanism that vibrates in a sinusoidal wave due to gear transmission, and the impact that the system receives was discussed by introducing the jerk, a first-time derivative of acceleration. Based on this basic modeling, an optimization problem with the change in the length of the crank and connecting rod constituting the four-bar linkage as a variable was finally established, and the process of deriving the solution was performed. The equation of motion was derived using the Newmark, direct integration, and the transient response obtained by applying a constant rotational force was compared. Unlike the transmission of rotation by gears, the deviation of the behavior of the four-bar mechanism follower increases, and the non-sinusoidal vibration changes according to the length ratio of the crank and connecting rod, so the effect on the playing process was analyzed.

Key Words - Mold oscillator, Structural dynamics, Non-sinusoidal waves, Vibration, Mold oscillation mark, Jerk

1. Introduction

In the steel industry, the casting process has significantly affected a dramatic increase in production. The continuous casting process is a process in which complex physical phenomena such as the solidification of molten steel, heat transfer by cooling, and powder inflow between mold and molten steel are challenging to analyze. However, many problems were supplemented with numerical solutions obtained through computer-based engineering models. Among them, the mold's vibration is the most significant factor that influences the casting process's speed. A mold vibrator is a device that introduces high-viscosity molten steel into a roll in a constant form, and various forms of vibration are added according to the speed of operation. Mold vibrators are generally electro-hydraulic servo devices and non-sinusoidal wave generators, which vibrate in a specific waveform by each mechanism. Each machine has advantages and disadvantages, but the most significant feature is that it applies to vibration by creating non-sinusoidal waves rather than vertical vibrations in the form of regular sine waves.

About the previous works, studies [1] and [2] of inducing non-sinusoidal vibration by applying control theory to an electro-hydraulic servo device as a mold vibration device were preceded based on fundamental ideas [3] and [4]; studies that generate non-sinusoidal waves by kinematic relationships without controlling the input signal have been the main focus. The content dealt with in this study is close to the latter case. A study was conducted to transmit non-constant rotational behavior using an elliptical gear [5] and generate non-sinusoidal waves using a Four-bar linkage mechanism [6]. In addition, studies [7-9] were conducted to identify dynamic characteristics by comparing non-sinusoidal and sinusoidal waves according to various environmental variables regarding casting speed, vibration frequency, and stroke.

Based on this fact, in this paper, we confirmed the relationship between the vibration mark on the molten steel surface generated by the mold vibration according to the waveform. While previous studies have experimentally approached how the vibration waveform affects the vibration mark, this study analyzed the behavior that occurs according to the shape of a four-bar mechanism applied to generate a non-sinusoidal wave. In particular, by introducing the concept of a jerk, a differential form of acceleration, an analysis was performed from the viewpoint of system maintenance, considering the impact applied to the mold vibrator, including suppression of vibration marks, which is a straightforward goal. In addition, based on the basic modeling mentioned above, the physical meaning was assigned to each behavior according to the shape of the four-bar linkage mechanism, and analysis was performed to find the optimal crank and connecting rod length ratio.

2. System Modeling

2.1 Four bar linkage

A four-bar linkage mechanism has a finite chain of 1 degree of freedom, and the number of links is 4 [10, 11]. A mechanism composed of links obtains various motions by changing the conditions of the links and is used in various industries. Generally, the four-bar linkage mechanism to transmit rotational force is divided into parallel and non-parallel manipulator mechanisms. In the case of a parallel manipulator, when the length of the two pairs of links is the same when the driving link rotates once, the driven link also rotates once, and the rotational speed is the same. Due to these characteristics, it was used to drive the wheels of locomotives. However, to generate a non-sinusoidal wave, as in this paper, irregular rotation must be applied at the driven shaft, even if a constant rotation is applied at the drive shaft. Non-parallel manipulator refers to a state in which the lengths of the paired links are different. Unlike the parallel manipulator, it has a rotational speed different from the driving shaft to obtain a different rotation. For mathematical expression, the kinematic picture of a general four-bar mechanism is shown in Fig. 1, and at this time, it has the constraint equations as Eqs. (1)-(4). Eqs (1) and (2) are obtained from the cosine law, and Eqs. (3) and (4) are obtained from the x and y direction constraints of the four-bar mechanism. By rearranging the above four equations with the trigonometric law and the derivative concerning time, the transfer ratio can be obtained, as shown in Eq. (5).

In the case of the four-bar linkage's transmission ratio, it can be seen that the value is different depending on the rotational displacement rather than a constant, so the value becomes complicated in terms of velocity and acceleration. When applying these kinematic characteristics to the equations of motion in this study, it was assumed that each link was considered a rigid body and that no friction existed in the rotary joint connecting the links.

$$r_p^{\,2} = a^2 + b^2 - 2ab\cos(180 - \theta_1 + \psi) \tag{1}$$

$$r_p^{\,2} = c^2 + d^2 - 2cd\cos(180 - \theta_2) \tag{2}$$

$$\sin(\psi) = \frac{c\sin(\theta_2) - a\sin(\theta_1)}{b} \tag{3}$$

$$\cos(\psi) = \frac{d + c\cos(\theta_2) - a\cos(\theta_1)}{b} \tag{4}$$

$$r = \frac{\omega_2}{\omega_1} = \frac{ad\sin(\theta_1) + ac\sin(\theta_1 - \theta_2)}{cd\sin(\theta_2) + ac\sin(\theta_1 - \theta_2)} \tag{5}$$

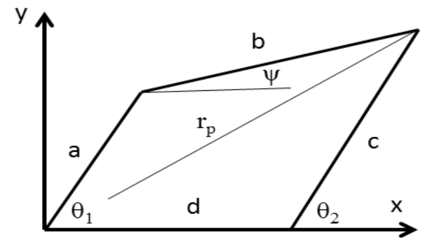

Fig. 1. Four bar linkage

2.2 Gear

Gears are widely used in parts that transmit the rotation and power of various machines [12]. The gear rotates at an accurate angular speed ratio through protrusion contact between the drive and driven gear. Due to the precise speed ratio, it transmits considerable power, has good efficiency, and can decelerate. However, it requires precision in production and has problems with noise and vibration. This study compared transmission by gears to objectively compare non-parallel manipulators that generate non-sinusoidal waves. As an assumption used in configuring the system, the transmission efficiency is one, and the gear phase difference due to the rotational moment of inertia and the elastic energy of the gear teeth does not occur was used. Like a non-parallel manipulator, it has a transmission ratio but has constant, so it is easy to grasp the behavior.

$$i = \frac{\omega_2}{\omega_1} = -\frac{R_1}{R_2} = -\frac{N_1}{N_2} \tag{6}$$

where ω, *i*, and *N* mean the rotational speed, radius, and number of teeth of each gear.

2.3 Up and down oscillation by the eccentric shaft

Cams are often used in systems that convert rotational motion into translational motion. In particular, in the case of an automobile engine, it is used to convert the reciprocating motion of the piston inside the cylinder into rotational motion. In the case of translational motion by cam, the path varies depending on the shape of the cam surface. In this study, an eccentric shaft that performs simple harmonic motion was constructed to express the vertical vibration by the eccentric shaft of the previous research, which is a part of the cam.

$$y = y_{eccen}\sin(\theta_2) \tag{7}$$

where y_{eccen} means the eccentric distance defined by the eccentric shaft, and θ_2 means the rotational displacement of the driven shaft in the rotation transmission system.

2.4 Equations of motion

In general, a mold vibration system including a non-parallel manipulator, a motor, a gear, a shaft, a coupling, a non-parallel manipulator, an eccentric shaft, a flask, and a leaf spring are placed in this order. This method of analyzing vibration by obtaining a complete dynamic model considering each flexibility is adopted. However, in this study, to directly grasp the sensitivity of the non-parallel manipulator's mechanism that determines the flask's non-sinusoidal vibration, a system was constructed with a drive shaft, a driven shaft, and an eccentric shaft. Flexibility, such as vibration and deformation experienced by each part, is used to identify vibration patterns in detailed design, so it needs to be discussed in the future based on this study. Fig. 2 shows a mold vibrator model with three components. The system's motion equation is arranged using the Lagrangian equation, and the assumption that includes kinetic energy and does not exist in the case of conservation energy was used. In addition, it was configured as an ideal system with no energy loss in transmission. Eqs. (8)-(12) enumerate the process of obtaining the equations of motion from system energy [13, 14].

$$T = \frac{1}{2}J_{driving}\left(\dot{\theta}_1\right)^2 + \frac{1}{2}J_{driven}\left(\dot{\theta}_2\right)^2 + \frac{1}{2}J_{eccentric}\left(\dot{\theta}_3\right)^2 + \frac{1}{2}J_{mold}\left(\dot{y}\right)^2 \tag{8}$$

$$V = 0 \tag{9}$$

$$L = T - V \tag{10}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j = Q_{ext}(t) \tag{11}$$

$$[M]\{\ddot{\vec{q}}\} + [C]\{\dot{\vec{q}}\} + [K]\{\vec{q}\} = \{Q_{ext}(t)\} \tag{12}$$

where T and V is the kinetic energy and potential energy of the system, j is 1, 2, 3, 4, Q_{ext} is external force, M, C, and K are mass, damping, and stiffness matrix, and q is generalized coordinates

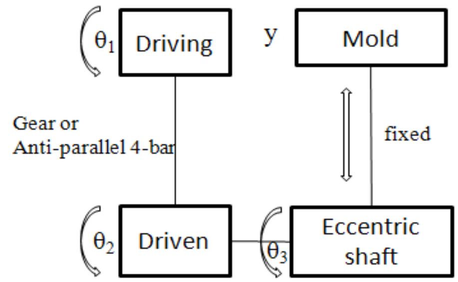

Fig. 2. Mathematical model of mold oscillation system

3. Numerical analysis and state variables

3.1 Numerical analysis

Several numerical methods exist to solve the vibration problem. The most commonly used method is the numerical integration method. Instead of obtaining a correct solution for the entire time, a solution that satisfies the equation of motion for a time interval specified by the user is received and applied to the next interval. [15-19] Finite difference, Runge-Kutta, Houbolt, Wilson, and Newmark are typical numerical integration methods. Using the Newmark method, this paper obtained the solution at $t_i=i\Delta t$ from the known solution at $t_{i-1}=(i-1)\Delta t$.

Newmark's method starts with the assumption that the acceleration varies linearly between two instants. At this time, displacement and velocity vectors in multiple degrees of freedom are expressed in Eqs. (13-14).

$$\vec{q}_{i+1} = \vec{q}_i + \Delta t\dot{\vec{q}}_i + \left[\left(\frac{1}{2} - \alpha\right)\ddot{\vec{q}}_i + \alpha\ddot{\vec{q}}_{i+1}\right](\Delta t)^2 \tag{11}$$

$$\dot{\vec{q}}_{i+1} = \dot{\vec{q}}_i + \left[(1-\beta)\ddot{\vec{q}}_i + \beta\ddot{\vec{q}}_{i+1}\right]\Delta t \tag{12}$$

In the case of the above displacement and velocity vectors, the stability and accuracy of the solution are determined according to the coefficients of α and β. In this paper, α = 1/6 and β = 1/2 were taken, and it was assumed that the acceleration changes linearly between time intervals. The displacement, velocity, and acceleration vectors and the external force term vector are obtained by substituting the initial conditions. Then, by substituting into the equation of motion, the new displacement, velocity, acceleration, and external force after Δt are sequentially obtained, and the final value is reached.

3.2 State variables

This paper's main analysis results are the velocity and jerk of the mold. The most challenging part of continuous casting is the vibration marks on the surface of the molten steel. Vibration marks are classified according to their sizes, and the deeper the vibration marks are, the more likely they are to generate cracks in the rolling process or final product. Accordingly, many studies have analyzed the causes of vibration marks, and the reason is attributed to the relationship between casting speed and the velocity of the mold. Fig. 3 shows an example of the relationship between the casting speed and the velocity of the mold. In a sinusoidal wave with a constant rotational speed, the vibrating rate of the mold also increases as the rotational speed increases. At this time, the vibrating velocity of the mold is lower than the casting speed, so it vibrates downward faster than molten steel. Molten steel comes down in the form. Even if the velocity of the mold returns to a higher value than the casting speed, grooves are created, and vibration marks are generated. At this time, the section where the velocity of the mold is lower than the casting speed is called Negative Strip Time, and the performance goal is to reduce the area in Fig. 3.

Second, jerk is a physical quantity that has yet to be dealt with in previous studies. In general, it is a physical quantity known as the differential value of acceleration, and when the change in acceleration is significant, it has an enormous jerk. Regarding the equation of motion, acceleration is a physical quantity determined by external force and mass and considerably influences the system's behaviour. However, in practice, what has a significant effect on the system is the amount by which the external force changes. When a specific external force is applied to the two-degree-of-freedom system, free vibration occurs at a distance equal to the corresponding displacement. However, when the external force has the same form as the natural frequency or the change amount is large, the vibration experienced by the system is irregular and reaches a critical point. This phenomenon should be considered because the same or different external force can be transmitted through the transmission device rather than simply applying a constant external force to the mold in this study. Based on this point of view, jerk is a measure of the change in applied force, and the instability inherent in the mold can be confirmed.

Based on the above facts, the velocity and the jerk of the mold were used as criteria to evaluate the vibration mark generation and stability of the mold vibration, respectively.

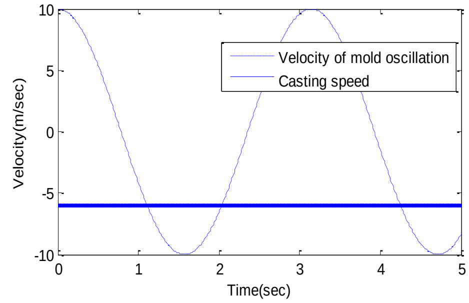

Fig. 3. Negative strip time along with velocity in sinusoidal oscillation

4. Results and discussions

4.1 Comparison of gear transmission and non-parallel manipulator transmission

Transmission of rotation through a non-parallel manipulator's mechanism, represented as Antiparallel in Figs. 4 and 5, differs from gear transmission by including the rotational speed term in the transmission ratio. Figs. 4 and 5 show the rotational behaviour of the driven shaft and the translational behaviour of the mold of the two models. In the case of the drive shaft, the same external force was applied to both models, resulting in the same diagram. On the other hand, in the rotational behaviour of the driven shaft, it can be confirmed that the aspect changes from displacement to jerk when delivering a non-parallel manipulator. When the two models are compared regarding the rotational speed of the driven shaft, it can be seen that the rotational speed is constantly increased in gear transmission. This phenomenon means that the rotation phenomenon generated in the drive shaft transmits the same tendency, only with a difference in magnitude according to the gear ratio. On the other hand, in a non-parallel manipulator, it can be confirmed that the average value rises as the rotational speed repeatedly increases and decreases. It can be seen that this phenomenon is consistent with the fact that the system has an unstable rotation phenomenon and the fact that peak points occur regularly in the acceleration and jerk curves.

This rotational phenomenon of the driven shaft is reflected in the translational motion of the mold frame by the eccentric shaft, as shown in Fig. 5. About the displacement diagrams of the two models; it can be seen that they are located at the same peak and trough simultaneously. However, there is a big difference in the route to the exact location. Regarding the velocity, the resulting two models are repeated according to the same frequency and have different peaks. When the negative strip section defined above is applied, it can be confirmed that the amplitude of the speed peak in the non-parallel manipulator decreases, and the overlapping part becomes smaller, which is advantageous in preventing the formation of vibration marks.

However, the vibration characteristics have very different aspects depending on the length ratio of the links constituting the non-parallel manipulator. When the lengths r_1 and r_2 of the links constituting the driving shaft and the driven shaft are the same, the length of the connecting rod is l. The r/l value is 0, and the flask vibration behaviour between the two models is the same. When the r/l value is close to 1, it can be confirmed that the speed peak point becomes more significant in the non-parallel manipulator, making it difficult to avoid the negative strip section. In addition, the generation unit of jerk also rises exponentially, resulting in instability in the system. Accordingly, confirming the need to establish and use the criteria for the optimal model of a non-parallel manipulator that generates non-sinusoidal waves is possible.

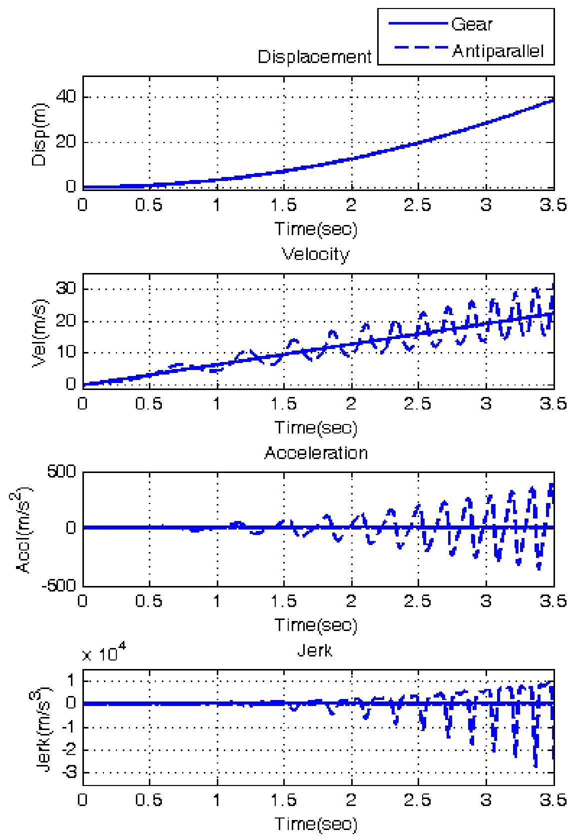

Fig. 4. Dynamic behaviors of the driven and eccentric shaft

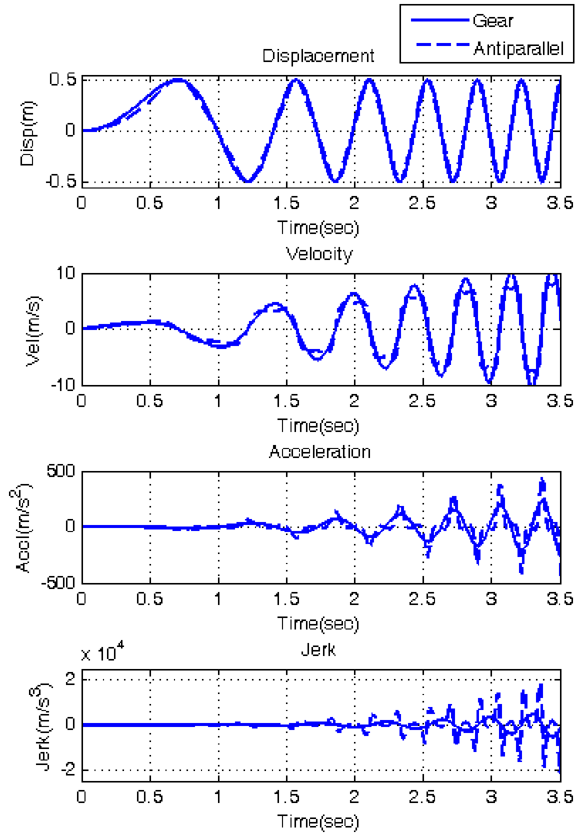

Fig. 5. Dynamic behaviors of the mold

4.2 Optimal model of the non-parallel manipulator

In this section, when the length of the links constituting the non-parallel manipulator is set as a variable, the difference between the vibration speed distribution of the mold and the standard value is averaged and compared. In the previous studies, the superiority of non-sinusoidal vibration was demonstrated by linking the negative strip area difference to forming vibration marks. In this study, the goal of isolating the mold's casting speed and velocity was set by arranging the peak point of the vibration speed to minimize the formation of vibration marks.

Fig. 6 shows the distribution of the velocity and average jerk value of the mold according to the r/l ratio. At this time, it was confirmed that the same vibration results were obtained when the lengths of the link of the drive shaft and the driven shaft and the length of the connecting rod were different, but the ratios were the same. As the r/l value approaches 1, the velocity and jerk increase exponentially. It can be seen that this adversely affects both vibration marks and system stability. On the other hand, the closer the value of r/l is to 0, the smaller the magnitude of the velocity and jerk. In general, the jerk has a phenomenon in which the size increases as the link length and the connecting rod becomes similar.

On the other hand, in the case of the velocity, a significant change appears in a specific section. Fig. 7 represents a section in which the sign of the difference has changed. In the area where the velocity value is negative, the average peak of the vibration speed made of gear transmission has a more significant value than the average speed peak of the non-parallel manipulator, and the difference is negative. As a result, the existing negative strip overlapping section is not created or reduced, giving a great advantage to forming vibration marks. Up to the case where the *r*/*l* value is approximately 0.4, the velocity shows the effect of being smaller than the standard, and it means that the velocity of the mold is the best at around 0.17. At this time, the irregularity of the curve generated in the *r*/*l* range of 0.12 to 0.14 will be discussed in the future on the effectiveness of selecting the reference value and whether it can play an influential role in forming vibration marks.

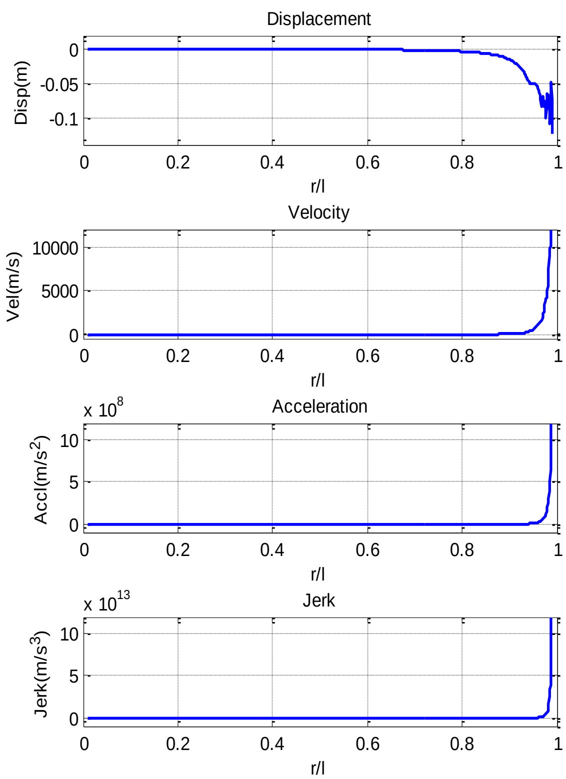

Fig. 6. Distributions of dynamic behaviors according *r*/*l*

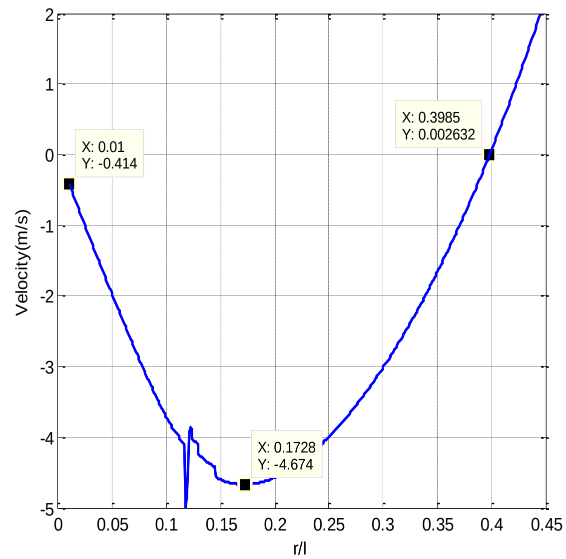

Fig. 7. Section of low velocity of mold oscillation

5. Conclusion

In this study, the transmission device that induces the vibration pattern of the mold vibrator was mathematically simulated, and the vibration pattern of the flask that appeared when rotation was transmitted using a non-equilibrium exercise device and gear was compared. The two transmission devices are essential factors that determine the vibration pattern of the mold frame and also act as factors that determine the formation of vibration marks in molten steel through overlapping comparisons between the speed distribution of the mold vibrator and the casting speed. Accordingly, in this study, the vibration of the flask with sinusoidal and non-sinusoidal waves was compared with physical quantities, including jerk, and a model of a non-parallel mechanism was presented to minimize the area in the situation where the casting speed and the vibration speed of the flask were unavoidably overlapped. In particular, the dynamic characteristics of the mold vibrator vary greatly depending on the ratio of the link and connect rod of the non-parallel manipulator. It was suggested that the *r*/*l* ratio should be within 0.4 to create an optimal vibration pattern.

In addition, unlike previous studies that focused on generating vibration marks, the jerk concept, which represents the differential of acceleration or the change in the overall external force received by the system, was introduced to show instability that may occur in the mold vibrator system. This result implies the possibility of being applied to the system maintenance part to control system vibration by comparing it with the external force applied to the existing system based on the jerk calculation that causes system failure.

The results obtained through the simple rigid system configuration can provide objective facts for measuring the sensitivity between each system parameter. Based on this, extending the model considering flexibility, such as deflection and deformation occurring in each part, and performing the dynamic analysis can help build a system that predicts vibration data and behavior in the field.

Acknowledgement

This work was supported by the Sunmoon University Research Grant of 2023.

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