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Research Article

Measurement and improvement of efficiency of Library by mathematical modelling and finding reliability

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ABSTRACT

Here in this paper, we have developed the mathematical model of the library server using Markov birth - death process assuming that library system server system is based on exponential distribution. The model so developed by victimisation Chapman Kolmogorov differential equation and is solved by using Mathematica. The solution so obtained is analysed for various rates of failures and repair. The finding so obtained are discussed with the concerned authorities of the library to boost the efficiency of the library.

KEY WORDS

Chapman Kolmogorov Differential Equation, Library Client Server, Library Mirror Server, Markov Process, Mathematica, Reliability

INTRODUCTION

In today's era we all witness a long boom in Information technology that has transformed the world astonishingly into a better place. Of course, not to forget the risks those are brought along. These risks have pushed us to go after the diagnostic approach and minimize the risks, keeping the output optimised. And this is what showed us the way to go for investigating and improving the reliability of the system. Reliability is a well chosen-word for measuring the efficiency of a system at a given time t. In the field of education, Library is the prime intellect reservoir of any academic Institute however small or large. Library servers have now-a-days been upgraded from just catalogue holders to a digital epitome of knowledge. So, their undisrupted functioning has became a subject of extreme importance. In this paper, we measure the efficiency of library by surveying and identifying the methodology or set-up so as to boost the reliability of the server system. A server yields the assistance in optimising the client's request, centralising the storage of data and securing the retained data. Redundancy dole out important role to maintain the backup in case of any data breaches. This system is composed of client server and mirror server. In the event of any glitches in client-server, the system is taken over by mirror-server, resulting in ceaseless operation or functioning. However, client-server takes back its position, once fixed.

LITERATURE REVIEW

The given literatures discussed the various ways to analysed reliability. In [2] Arora et al. have performed the "System Analysis and maintenance management of the coal handling system in a paper plant". In [3] Kumar et al. have run the "Reliability analysis of the Feeding System in the Paper Industry". In [5] Kumar et al. have done "Performance evaluation and availability Analysis of ammonia synthesis unit in a fertilizer plant". In [6] Khanduja et al. discussed the "Availability analysis of bleaching system of a paper plant". In [8] Kumar et al. have analysed the "Performance Modelling and Simulated Availability of Shell Gasification and Carbon Recovery Unit of Urea Plant". In [10] Jindal et al. performed the "Analysis of the reliability of the Butter-oil Prrocessing plant using CAS Mathematica and Maxima". In [12] Lindemann et al. have used "Numerical methods for reliability evaluation of markov closed fault-tolerant systems". In [13] Modgil et al. gave study about "Performance modelling and availability analysis of shoe upper manufacturing unit". In [16] Garg et al. have analysed the Mathematical modelling and performance analysis of combed yarn production system. In [17] Shakuntla et al. have performed Reliability analysis of polytube industry using supplementary variable technique.

SERVER SYSTEM

System description

The system under study comprises of client server unit 'CS' and mirror server unit 'MS'. Standby state comes into action when either CS or MS do not execute the designated commands. And the working of the system ceases once either of the component server stops working completely or both of them simultaneously unable to execute the designated commands.

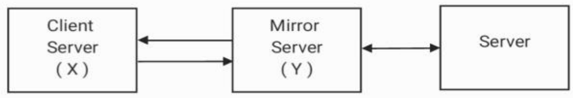
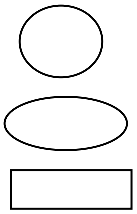


FIG. 1

Notations and Assumptions

Notations:



: Full capacity state.

: Reduced capacity state.

: Failed state.

- X, Y : denotes that unit CS and MS are in full capacity state respectively.
- X_s, Y_s : denotes that unit CS and MS are in reduced capacity state respectively.
- x, y : denotes that unit CS and MS are in failed state respectively.
- x_1, x_2 : mean failure rates of CS and MS respectively, from full working state to reduced state.
- x_3, x_4 : mean failure rates of CS and MS respectively, from reduced state to failed state.
- y_1, y_2 : mean repair rates of CS and MS respectively, from reduced state to full working state.
- y_3, y_4 : mean repair rates of CS and MS respectively, from failed state to full working state.
- $P_i(t)(i = 1,2,3,4,5)$: probability that the system is in specific " i^{th} " state at any time t.
- $P_i'(t)(i = 1,2,3,4,5)$: derivative with respect to t.

Assumptions:

- Mean failure and repair rates are independent and constant throughout.
- System remain ceaseless in reduced state.
- Server units cannot have a simultaneous failure.
- Each server unit has its own repairing facility.
- After repairing, the server units are as good as new in performance.
- Failure and repair follow exponential distribution.

MATHEMATICAL MODELING OF THE SYSTEM

We can see the involved viable states corresponding to the mathematical model of Library Server System, in the Transition diagram (FIG 2) given below.

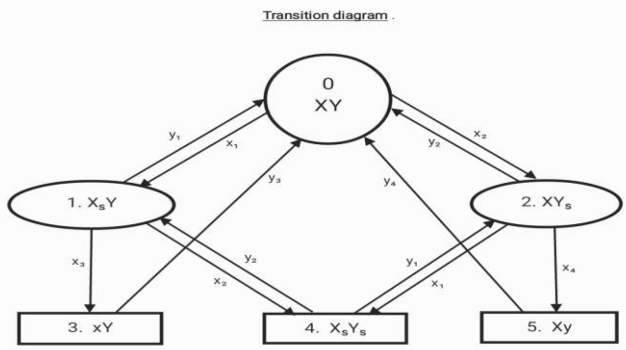


FIG. 2

Transient state

The accompanying system of differential equations with initial condition for each state can be written as follows:

$$\left[\frac{d}{dt} + x_1 + x_2\right] P_0(t) = y_1P_1(t) + y_2P_2(t) + y_3P_3(t) + y_4P_4(t) \quad (1)$$

$$\left[\frac{d}{dt} + x_2 + x_3 + y_1\right] P_1(t) = x_1P_0(t) + y_2P_4(t) \quad \dots (2)$$

$$\left[\frac{d}{dt} + x_1 + x_4 + y_2\right] P_2(t) = x_2P_0(t) + x_4P_5(t) + x_1P_4(t) \quad \dots (3)$$

$$\left[\frac{d}{dt} + y_3\right] P_3(t) = x_3P_1(t) \quad \dots (4)$$

$$\left[\frac{d}{dt} + y_1 + y_2\right] P_4(t) = x_2P_1(t) + x_1P_2(t) \quad \dots (5)$$

$$\left[\frac{d}{dt} + y_4\right] P_5(t) = x_4P_2(t) \quad \dots (6)$$

With initial conditions at time $t = 0$

$$P_i(t) = \begin{cases} 1 & \text{for } i = 0 \\ 0 & \text{for } i \neq 0 \end{cases}$$

Using Computer Algebra System Mathematica, the above of equations are solved under real working conditions at time t. Hence the reliability R(t) of the system is given as:

$$R(t) = P0(t) + P1(t) + P2(t) \quad \dots (7)$$

Where,

$P_0(t)$ gives the probability of full working state 0.

$P_1(t)$ gives the probability of reduced working state 1.

$P_2(t)$ gives the probability of reduced working state 2.

After solving resulting model, the values of working states P_0, P_1, P_2 P_3 P_4 and P_5 at a time t, are obtained as follows:

$$P_0(t) = -[0.023990370644286076e^{-0.54t}(1.e^{0.34976830452243385t} + 3.40726716122701e^{0.3951342549980241t} + 1.909099038740512e^{0.4418165130185517 4t} + 4.661287752030304e^{0.48513133987897306t} + 0.5414930236131562e^{0.4913699666887976t} + 30.164244032696587e^{0.5367796208932196t})]$$

$$P_1(t) = [-0.05528845555008991e^{-0.54t}(1.e^{0.34976830452243385t} - 1.1561337172449813e^{0.3951342549980241t} + 1.9075651061495396e^{0.4418165130185517 4t} - 0.4252196961002806e^{0.48513133987897306t} - 0.034512325273643195e^{0.4913699666887976t} - 1.2916993675306339e^{0.5367796208932196t})]$$

$$P_2(t) = [-0.013747625908532735e^{-0.54t}(1.e^{0.34976830452243385t} + 9.959337477120462\, e^{0.3951342549980241t} - 0.969383277446213e^{0.4418165130185517 4t} - 1.064267593411189e^{0.48513133987897306t} + 0.09060990759756882e^{0.4913699666887976t} - 9.01629651386063e^{0.5367796208932196t})]$$

$$P_3(t) = [0.0039426504373210 59e^{-0.54t}(1.e^{0.34976830452243385t} - 1.7090108907558617e^{0.3951342549980241t} + 5.55171710947751e^{0.4418165130185517 4t} - 12.247574784912405e^{0.48513133987897306t} + 3.532729611290014e^{0.4913699666887976t} + 3.872138954900745e^{0.5367796208932196t})]$$

$$P_4(t) = [0.03090213711675108e^{-0.54t}(1.e^{0.34976830452243385t} - 0.5720144970575687e^{0.3951342549980241t} - 1.234079939922848e^{0.4418165130185517 4t} + 0.2097130290694348e^{0.48513133987897306t} + 0.008206082427793587e^{0.4913699666887976t} + 0.5881753254831884e^{0.5367796208932196t})]$$

$$P_5(t) = [0.0019607016604505207e^{-0.54t}(1.e^{0.34976830452243385t} + 14.722013517321502e^{0.3951342549980241t} - 2.8212624091780967e^{0.4418165130185517 4t} - 30.65402910778671e^{0.48513133987897306t} - 9.274956152857118e^{0.4913699666887976t} + 27.028234152500417e^{0.5367796208932196t})]$$

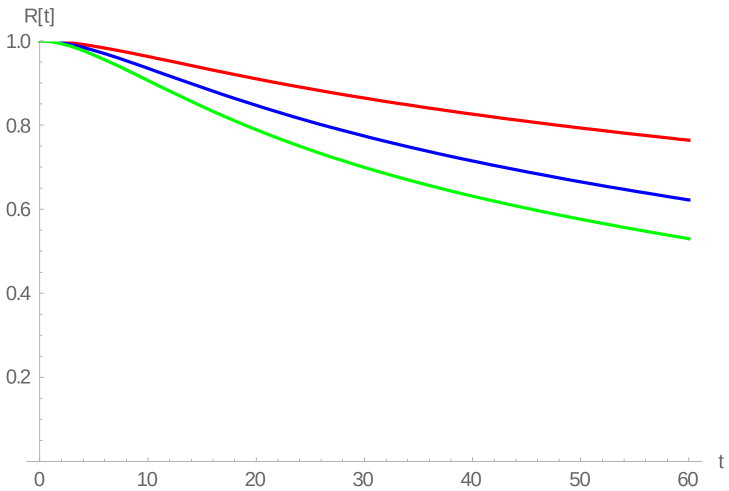
Analysis of the Reliability of the system

We calculate the reliability of the system by victimisation Eq. (7), and analysed the impact of variation in failure and repair rates on reliability of the system. Tables 1, 2 and 3 briefly explain the variation in reliability with graphical plotting.

When b_1=b_2=0.050

In Table-1 ***(Effect of failure and repair rates of subsystem on the reliability of the system)*** the reliability of the system is computed by varying their values as x_1 = x_3=0.01,0.02,0.03 and remaining constants fixed as: $x_2 = 0.02; x_4 = 0.02; y_1 = 0.05; y_2 = 0.1; y_3 = 0.05; y_4 = 0.05$;. we observe that the reliability of the system decreases with increase in failure rate and also with passage of time.

R(t) → / t ↓	$x_1 = x_3 = 0.01$	$x_1 = x_3 = 0.02$	$x_1 = x_3 = 0.03$
0	1	1	1
5	0.987507	0.977492	0.96653
10	0.9629	0.934739	0.90576
15	0.935888	0.889314	0.844161
20	0.909883	0.846704	0.788843
25	0.885886	0.808184	0.740763
30	0.864003	0.773638	0.699117
35	0.844031	0.742554	0.662759
40	0.825692	0.714365	0.630627
45	0.808711	0.688566	0.601852
50	0.792851	0.664732	0.575748
55	0.777912	0.642524	0.551791
60	0.763733	0.621671	0.529583



Note: 'Red' for x_1 =x2 = 0.01, 'Blue' for x1 = x2 = 0.02, 'Green' for x1 = x3 = 0.03

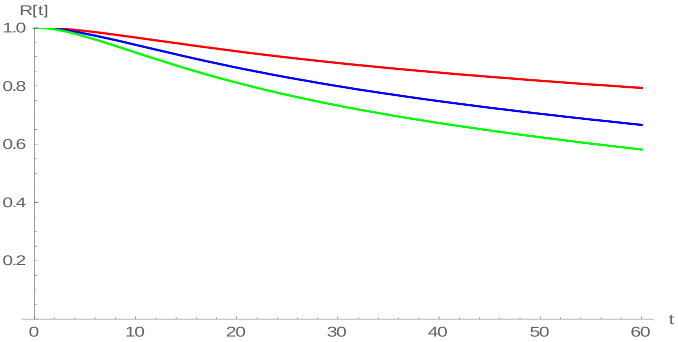
Graph 1

When y_1 = y_2 = 0.060

In Table-2 ***(Effect of failure and repair rates of subsystem on the reliability of the system)*** the reliability of the system is computed by varying their values as x_1 = x_3=0.01,0.02,0.03 and remaining constants fixed as: $x_2 = 0.02; x_4 = 0.02; y_1 = 0.06; y_2 = 0.1; y_3 = 0.06; y_4 = 0.05$;. we observe that the reliability of the system decreases with increase in failure rate and also with passage of time.

At same time if we analyse Table -1 and Table -2, we observe that reliability of the system increases with increase in repair rate.

R(t) → t ↓	$x_1 = x_3 = 0.01$	$x_1 = x_3 = 0.02$	$x_1 = x_3 = 0.03$
0	1	1	1
5	0.988491	0.979424	0.969374
10	0.966069	0.940831	0.914542
15	0.941752	0.90034	0.859709
20	0.918623	0.862761	0.810983
25	0.897523	0.829065	0.768918
30	0.87848	0.799005	0.732575
35	0.861253	0.772021	0.700799
40	0.845546	0.747544	0.672577
45	0.831081	0.725085	0.64711
50	0.817618	0.70425	0.62379
55	0.804964	0.684731	0.602166
60	0.792963	0.66629	0.581908



Note: 'Red' for x_1 =x2 = 0.01, 'Blue' for x1 = x2 = 0.02, 'Green' for x1 = x3 = 0.03

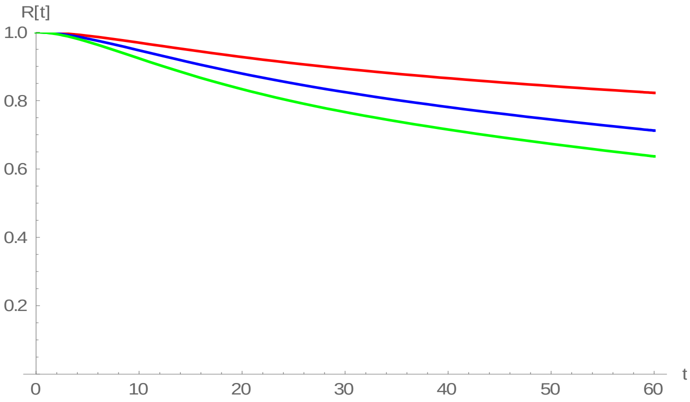
Graph 2

When y_1=y_2=0.070

In Table-3 (Effect of failure and repair rates of subsystem on the reliability of the system) the reliability of the system is computed by varying their values as a_1=a_3=0.01,0.02,0.03 and remaining constants fixed as: $x_2 = 0.02; x_4 = 0.02; y_1 = 0.07; y_2 = 0.1; y_3 = 0.07; y_4 = 0.05;$. we observe that the reliability of the system decreases with increase in failure rate and also with passage of time.

If we analyse Table -1, Table - 2 and Table -3 altogether, we observe that reliability of the system increases with increase in repair rate, decreases with increase in failure rate and also decreases with passage of time.

R(t) → t ↓	$x_1 = x_3 = 0.01$	$x_1 = x_3 = 0.02$	$x_1 = x_3 = 0.03$
0	1	1	1
5	0.989475	0.981356	0.972218
10	0.969237	0.946924	0.923329
15	0.947617	0.911374	0.875278
20	0.927366	0.878846	0.833189
25	0.909168	0.850011	0.797223
30	0.892976	0.824501	0.766323
35	0.878513	0.801718	0.739335
40	0.865471	0.781095	0.715302
45	0.853566	0.762166	0.693494
50	0.842564	0.744566	0.673379
55	0.832276	0.728018	0.654574
60	0.822554	0.712314	0.636806



Note: 'Red' for x_1 =x2 = 0.01, 'Blue' for x1 = x2 = 0.02, 'Green' for x1 = x3 = 0.03

Graph – 3

DISCUSSION AND CONCLUSION

The analysis of library server can help in finding the reliability of client and mirror server by using CAS Mathematica. The variation of reliability with change in failure rates (x_i) and repair rates (y_i) of unit A and unit B are shown above by Table-1, Table-2 & Table-3. We observed that reliability of the system increase with increase in repair rate and vice versa for transient states. The findings of the paper will be useful for library workers, working on server. Mathematica is method for better accuracy. Values given by Mathematica are exact values not approximates. Also, Mathematica gives individual values of P_0, P_1, P_2, P_3, P_4 even for complex structures at point t which is not possible to determine by other methods for complex models. P_0, P_1, P_2, P_3, P_4 are continuous function of time so R(t) is also continuous function of time, not discrete so by victimisation this method, we can solve system for transient state also not only for steady state. This software not only gives accuracy in results but also save computation time.

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