

Measurement Model

- It depicts how the observed (measured, manifest, indicators) variables represent constructs.
- Permits the assessment of construct validity.
- No single indicator can completely represent a construct (at least three helps in model identification).
- Uses the technique of Confirmatory Factor Analysis (CFA)
 - It seeks to confirm if the number of factors (or constructs) and the loadings of observed (indicator) variables on them conform the expectations of theory.
 - Tests the hypothesis on relationships b/w observed variables and their underlying latent constructs.

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Structural Model

- It shows how the constructs are interrelated to each other, often with multiple dependence relationships.
- It specifies whether a relationship exists or does not exist.
- If a relationship is hypothesized by the theory, then an arrow is drawn
- Measurement theory specifies how the constructs are represented; structural theory posits how the constructs are interrelated

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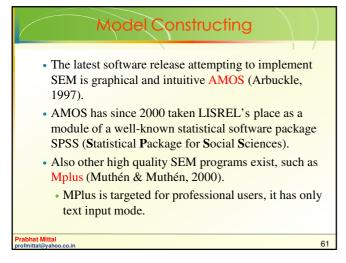
Model Constructing

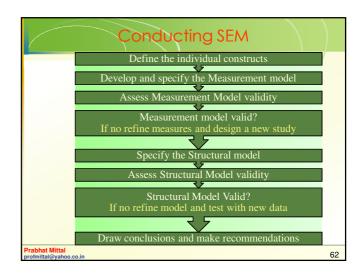
- One of the most well known covariance structure models is called LISREL (LInear Structural RELationships) or Jöreskog-Keesling-Wiley –model.
- LISREL is also a name of the software (Jöreskog et al., 1979), which is later demonstrated in this presentation to analyze a latent variable model.
- The other approach in this study field is Bentler-Weeks model (Bentler et al., 1980) and EQS -software (Bentler, 1995).

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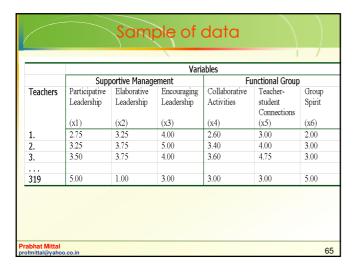


Define the Individual constructs Specific constructs, how each construct will be defined and measured, and the interrelationships among constructs must all be specified based on theory Prabhat Mittal professional pro

		Variable Des	scription
	Item	Summary variable	Sample statement
ayement	X1	Participative Leadership	It is easy to be touch with the leader of the training programme.
Supportive Marregement	X2	Elaborative Leadership	This organization improves it's members professional development.
Support	X3	Encouraging Leadership	My superior appreciates my work.
	X4	Collaborative Activities	My teacher colleagues give me help when I need it.
unctional Group	X5	Teacher – Student Connections	Atmosphere on my lectures is pleasant and spontaneous.
Z Z	X6	Group Spirit	The whole working community co-operates effectively.

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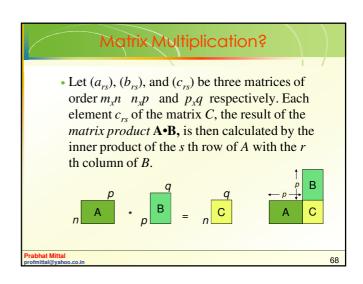


	Covariance Matrix								
	Su	pportive Mana	gement	Functional Group					
	X1	X2	Х3	X4	X5	X6			
K 1	.734	.343	.438	.220	.104	.275			
(2	.343	.668	.467	.234	.037	.307			
(3	.438	.467	.938	.306	.165	.391			
(4	.220	.234	.306	.459	.182	.308			
(5	.104	.037	.165	.182	.387	.114			
(6	.275	.307	.391	.308	.114	.552			

What is Covariance matrix?

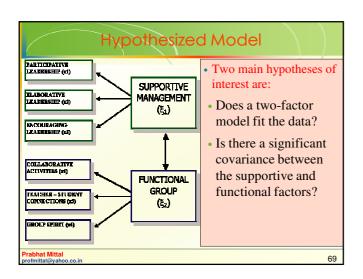
- Scatter, covariance, and correlation matrix form the basis of a multivariate method.
- The correlation and the covariance matrix are also often used for a first inspection of relationships among the variables of a multivariate data set.
- ${}^{\bullet}$ All of these matrices are calculated using the matrix multiplication (A ${}^{\cdot}$ B).
- The only difference between them is how the data is scaled before the matrix multiplication is executed:
 - scatter: no scaling
 - covariance: mean of each variable is subtracted before multiplication
 - correlation: each variable is standardized (mean subtracted, then divided by standard deviation)

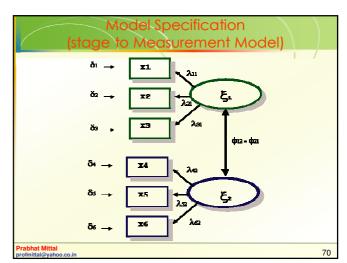
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Model Specification

• The relationships for this part of the measurement model can now be specified in a set of *factor equations* in a scalar form:

$$\begin{array}{ll} x_1 = \lambda_{11} \xi_1 + \delta_1 & \quad x_2 = \lambda_{21} \xi_1 + \delta_2 \\ x_3 = \lambda_{31} \xi_1 + \delta_3 & \quad x_4 = \lambda_{42} \xi_2 + \delta_4 \\ x_5 = \lambda_{52} \xi_2 + \delta_5 & \quad x_6 = \lambda_{62} \xi_2 + \delta_6 \end{array}$$

- δ_i is the residual variable (error) which is the unique factor affecting x_i .
- λ_{ij} is the loading of the observed (measured) variables x_i on the latent factor ξ_i .
- Φ is the correlation between constructs

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Model Specification

- In the earlier case, total of 13 parameters need to be estimated
 - Six loading estimates, Six error estimates and One constructs correlation

Note: For each relation that is not specified, the parameter is fixed and set at zero.

We should set the scale of latent construct (as it is not observed directly) using the following options:

- 1. One of the factor loadings can be fixed, generally equal to one
- 2. The construct variance can be fixed, generally to one.

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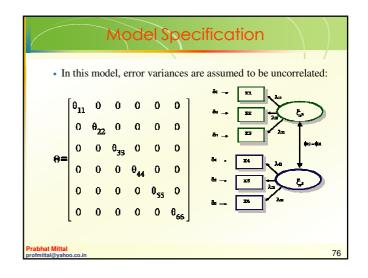
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2. Model Specification

- Most of the calculations are performed as matrix computations because SEM is based on covariance matrices.
 - To translate equation into a more matrix friendly form,

$$\begin{array}{l} x_1 = \lambda_{11}\xi_1 + 0\xi_2 + \delta_1 \\ x_2 = \lambda_{21}\xi_1 + 0\xi_2 + \delta_2 \\ x_3 = \lambda_{31}\xi_1 + 0\xi_2 + \delta_3 \\ x_4 = 0\xi_1 + \lambda_{42}\xi_2 + \delta_4 \\ x_5 = 0\xi_1 + \lambda_{52}\xi_2 + \delta_5 \\ x_6 = 0\xi_2 + \lambda_{62}\xi_2 + \delta_6 \end{array}$$
 Equivalent to
$$x = \Lambda_x \xi + \delta$$

$$\begin{array}{c|c} \text{Model Specification} \\ \hline & x = \Lambda_x \xi + \delta \\ \hline \begin{pmatrix} x_1 & 0 \\ x_2 & \lambda_{21} & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & 0 \\ 0 & \lambda_{42} \\ x_5 & 0 & \lambda_{52} \\ x_6 & 0 & \lambda_{62} \\ \hline \end{pmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \hline \end{bmatrix}$$



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Model Specification

- Because the factor equations cannot be directly estimated, the covariance structure of the model is examined.
- Matrix Σ contains the structure of covariances among the observed variables after multiplying equation by its transpose Σ = E(xx')

and taking expectations

 $\Sigma = \mathrm{E}[(\Lambda \xi + \delta) \; (\Lambda \xi + \delta)'] = \mathrm{E}[(\Lambda \xi + \delta) \; (\xi' \Lambda' + \delta')]$

 $= \mathrm{E}[\Lambda \xi \xi' \Lambda' + \Lambda \xi \delta' + \delta \xi' \Lambda' + \delta \delta']$

 $\Sigma = \mathrm{E}[\Lambda \xi \xi' \Lambda'] + \mathrm{E}[\Lambda \xi \delta'] + \mathrm{E}[\delta \xi' \Lambda'] + \mathrm{E}[\delta \delta']$

 $\Sigma = \Lambda E[\xi \xi'] \Lambda' + \Lambda E[\xi \delta'] + E[\delta \xi'] \Lambda' + E[\delta \delta']$ Since the values of the parameters in matrix Λ are constant

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Model Specification

 Since E[ξξ'] = Φ, E[δδ'] = Θ, and δ and ξ are uncorrelated, previous equation can be simplified to covariance equation:

 $\Sigma = \Lambda \Phi \Lambda' + \Theta$

- The left side of the equation contains the number of unique elements q(q+1)/2 in matrix Σ .
 - q denotes the no. of observed & 's' no. of latent variables
- The right side contains qs + s(s+1)/2 + q(q+1)/2 or (s+q)(s+q+1)/2) unknown parameters from the matrices Λ , Φ , and Θ .
- Unknown parameters have been tied to the population variances and covariances among the observed variables which can be directly estimated with sample data.

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Model Identification

- Identification is a theoretical property of a model, which depends neither on data or estimation.
 - When our model is identified we obtain unique estimates of the parameters.
- "Attempts to estimate models that are not identified result in arbitrary estimates of the parameters." (Long, 1983, p. 35.)

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Model Identification

- A model is identified if it is possible to solve the covariance equation $\Sigma = \Lambda \Phi \Lambda' + \Theta$ for the parameters in Λ , Φ and Θ .
 - Estimation assumes that model is identified.
- There are three conditions for identification:
 - necessary conditions, which are essential but not sufficient,
 - sufficient conditions, which if met imply that model is identified but if not met do not imply opposite (unidentified),
 - necessary and sufficient conditions.

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Model Identification

- Necessary condition is simple to test since it relates the number of independent covariance equations to the number of independent parameters.
- Covariance equation contains q(q+1)/2 independent equations and qs + s(s+1)/2 + q(q+1)/2 or (s+q)(s+q+1)/2) possible independent parameters in Λ , Φ and Θ .
 - Number of independent, unconstrained parameters of the model must be less than or equal to q(q+1)/2.
 - Note that this number is the sum of all the unique covariance q(q-1)/2 and all the variances 'q'

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Model Identification

- We have six observed variables and, thus, 6(6+1)/2 =21 distinct variances and co-variances in Σ .
 - There are 15 independent parameters:

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 0 \\ 0 & 4 \\ 0 & 5 \\ 0 & 6 \end{bmatrix} \qquad \bullet \bullet \bullet \begin{bmatrix} 7 & 8 \\ 8 & 9 \end{bmatrix} \qquad \bullet \bullet \bullet \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 11 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13 & 0 & 0 \\ 0 & 0 & 0 & 0 & 14 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 \end{bmatrix}$$

Model Identification

• Since the number of independent parameters is smaller than the independent covariance equations (15<21), the necessary condition for identification is satisfied.

Levels of Identification

Identification – degree to which there are a positive dof. And there is a sufficient number of equations to 'solve for' each of the coefficients (unknowns) to be estimated.

Three possible levels:

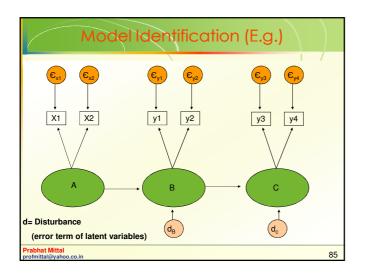
- Under-identified negative dof. (model cannot be fit)
- **Just-Identified** zero dof. (no. of equations = no. of estimated coefficients)
- Over-identified positive dof. (more equations than no. of estimated coefficients)

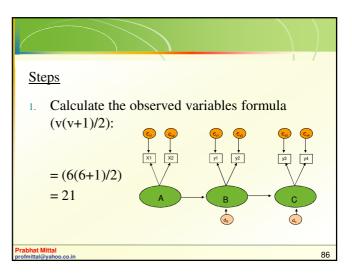
Objective: achieve acceptable fit with largest number of dof. Formula to compute (dof) – (Hair, page 608)

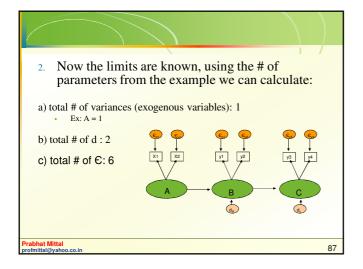
df = (s+q)(s+q+1)/2 – no. of estimated coefficients

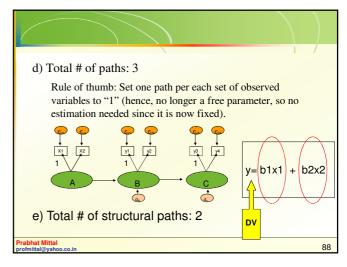
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- 3. Now we must add up all the values: 1+2+6+3+2=14
- Please note that our task is much eased since AMOS will tell you if you have the correct number of parameters.
 - It will give you an error, or not run at all if it is underidentified.
 - NB: if your model is based on theory, identification should not be an encountered problem.

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Model Identification

- The most effective way to demonstrate that a model is identified is to show that each of the parameters can be solved in terms of the population variances and covariances of the observed variables.
 - Solving covariance equations is timeconsuming and there are other 'recipe-like' solutions.

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Model Identification

- We gain constantly an identified model if
 - each observed variable in the model measures only one latent factor and
 - factor scale is fixed (Figure 6) or one observed variable per factor is fixed (Figure 7). (Jöreskog et al., 1979, pp. 196-197; 1984.)

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Model Estimation

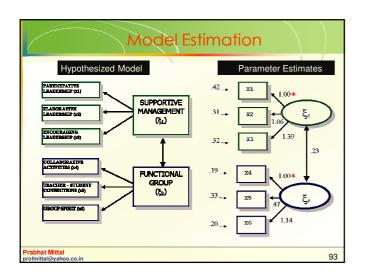
- When identification is approved, estimation can proceed.
- If the observed variables are normal and linear and there are more than 200 observations, Maximum Likelihood estimation is applicable.
- Large samples are required when the communalities (variance of a measured variable that is explained by the construct on which it loads) become smaller than 0.5.
- Note: SEM models Constructs <= 5, each with indicators > 3, and communalities >0.5 preferably Sample size >200.
- SEM models Constructs <= 5, each with indicators <= 3, and communalities <0.5 preferably Sample size >300.

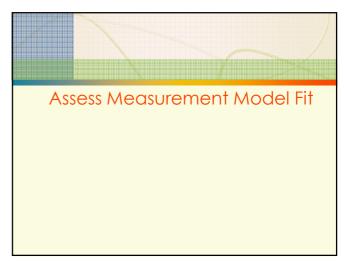
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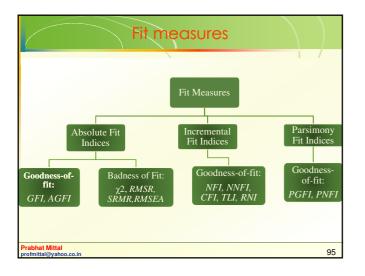
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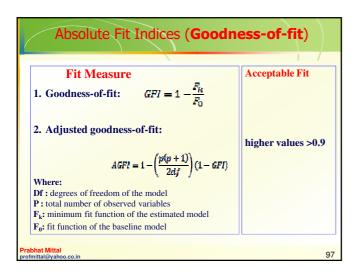
Fit measures

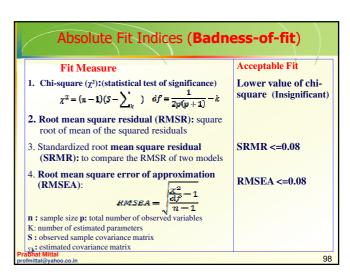
- Absolute fit Indices each model is evaluated independently of other
 possible models. These indices directly measure how well the specified
 model reproduces the observed or sample data.
 - Goodness-of-fit indices indicate how well the specified model fits the observed or sample data, and so higher values are desired.
 - Badness-of-fit indices measure error or deviation in some form and so lower values on these indices are desirable.
- Incremental fit indices evaluate how well the specified model fits the sample data relative to some alternative model related to a baseline model.
- Parsimony fit indices are designed to assess fit in relation to model complexity and are useful in comparing models. (*ratio of df used by the model to the total df*)

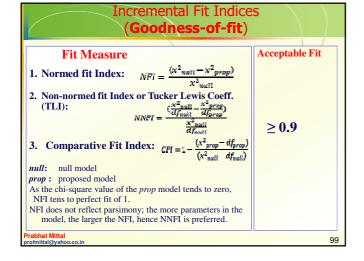
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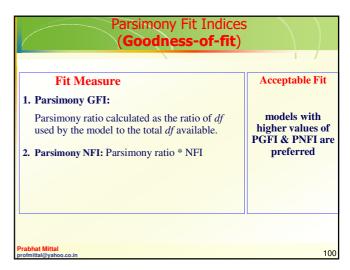
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Conclusions

- *CFI* and *RMSEA* are among the measures least affected by sample size and quite popular in use.
- It is a good practice to always report chi-square value with the associated *df*.
- It is a good practice to use at least one from each i.e. goodness-of-fit, one absolute badness-of-fit and one incremental fit measure.
- If models of different complexities are taken, it is good to use one measure from parsimony fit index.

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Measurement Model Reliability, validity and modification

• Composite Reliability (CR) • CR ≥ 0.7 are considered good • CR ≥ 0.7 are considered good • Completely standardized factor loading Error variance • No. of indicators or observed variables • Convergent Validity: it measures the extent to which the scale correlates positively with other measures of the same construct. Size of the factor loadings determine CV. High factor loadings should be significant and higher than 0.7. • AVE Average Variance Extracted Variance explained by latent construct • AVE should be ≥ 0.50 • Prabhat Mittal probmittal Systopoco.in

Lack of Validity: Diagnosing Problems

- Path estimates or loadings.
 - Standardized loadings should be in range -1 to 1.
 - Loadings on the indicators should be statistically significant preferably above 0.7. A non-significant can be dropped
 - Sign of the loadings should be in the direction hypothesized by the theory.
- Standardized residuals: observed covariance on the sample data estimated covariance terms divided by the standard error.
 - Absolute values of SR > 4 are problematic
- Specification approach empirical approach to find a better-fitting model.
- Generally not recommended.
- Note that all adjustments/modifications in upto 10% of the observed variables is permissible. Else you must modify the measurement theory and specify a new measurement model.

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Model Modification

- If indices indicate a poor fit, you can do posthoc modifications to see if it is possible to achieve fit.
 - Omission of variables,
 - Dropping non-significant paths,
 - · Adding significant paths.
- Caveat: SEM is a knowledge based testing statistical tool. Therefore, applying a post-hoc modification can be a poor practice in theory.

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Model Modification-II

- Must remember that it is unreasonable to expect a structural model to fit perfectly.
 - A structural model with linear relations is only an approximation and the world is unlikely to be linear.
 - So instead of asking "Does the model fit perfectly?", you must ask "Does it fit well enough to be a useful approximation of reality and a reasonable explanation of the trends in the data?".

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Model Modification-III

- Simply because a model fits well, it does not prove that the model is correct.
 - Fit indicates the that you are on the right track, however you must acknowledge the possibility that it could also be wrong, or that another could be even better.
 - Therefore, it is a fallacy to affirm proof.

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Relationship of SEM to Other Multivariate Techniques

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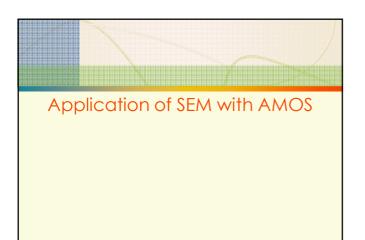
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- SEM is a dependence technique similar to other multivariate dependence techniques such as Multiple Regression. The endogenous construct is the dependent variable and the constructs with arrows pointing to the endogenous construct are the independent variables. However
 - Dependent in one relationship may become the independent in another relationship unlike Multiple regression.
 - All equations are estimated simultaneously unlike multiple regression.
 - Similar to MANOVA if categorical variables are used in SEM.

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- Measurement model of SEM is similar to factor analysis.
- Concepts of correlation and covariance are common in both techniques.
- SEM requires the specification of the measurement model. The loadings are estimated only for the specified relationships and all other loadings are assumed to be zero. Thus SEM is a confirmatory technique.
- In contrast, factor analysis is an explanatory technique EFA. It identifies factors that explain the correlations among a set of variables. Every variable has a loading on every factor extracted which are contained in the factor matrix.

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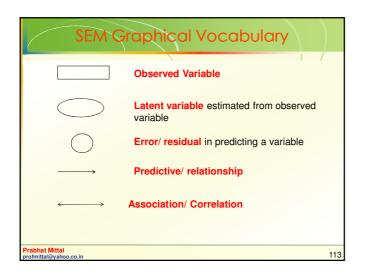
Technology Acceptance Model: TAM

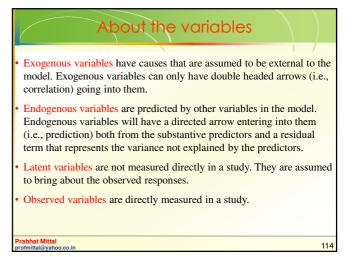
- To predict individuals' reactions to an information technology.
 - A structural model with linear relations is only an approximation and the world is unlikely to be linear.
 - So instead of asking "Does the model fit perfectly?", you must ask "Does it fit well enough to be a useful approximation of reality and a reasonable explanation of the trends in the data?".

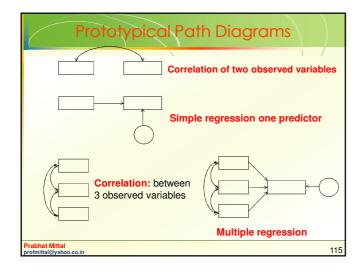
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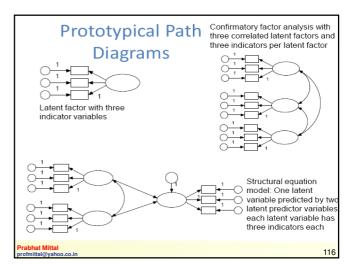
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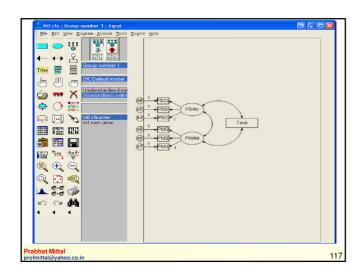


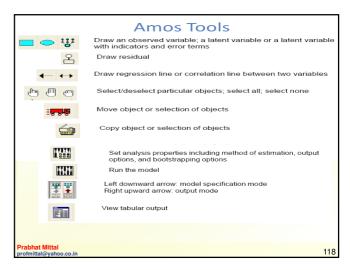


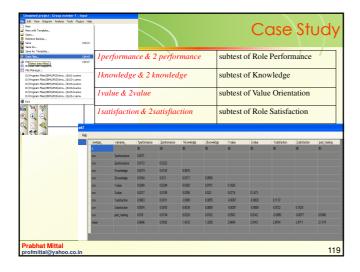


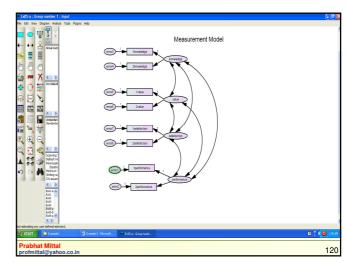


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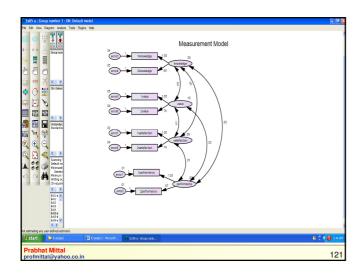


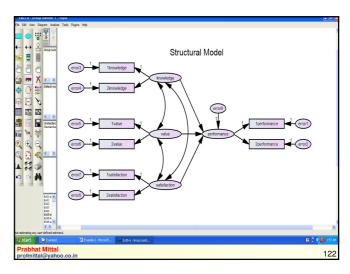


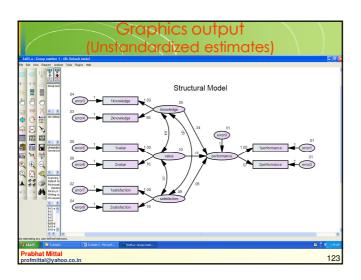


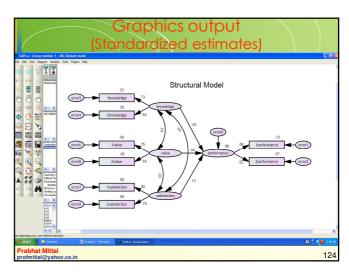


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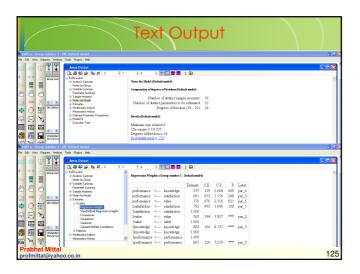


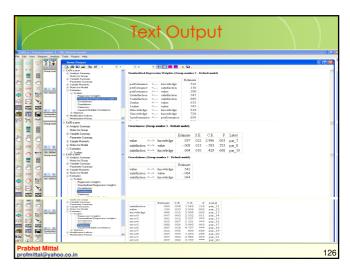






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Advantages of SEM

- Software is very user friendly.
- Allows models with latent variables.
- Studies complex multivariate relationships that are closer to reality (vs. exploratory methods).
- Compensates for lack of perfect reliability in measurement scales (therefore reveals true relationship between variables).
- Due to necessary a priori hypothesis, yielded model provides stronger evidence.

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Limitations of SEM

- If there is not enough theoretical background, the model WILL suffer
- The model is only as good as the validated tests used in the experiment to measure the observed variables.

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